3. The Concept of Length

TIMS Tutor

3.1 Units

In dealing with scientific problems, we must often know among other things the location of an object, the distance between objects, how tall or wide an object is, how fast it is moving, etc. Central to all of these ideas is the concept of length. In this part of the TIMS program we shall go into the various disguises in which length can appear and discuss a wide variety of experiments involving length measurement.

First, however, we must go into the messy problem of units. The kinds of units chosen for length were for a long time quite arbitrary. The cubit, used by several ancient civilizations, was the length of a forearm between the tip of the middle finger and the elbow. The fathom was the width of a viking sailor's embrace. (Fathom that!)

The foot was a convenient length defined as the length of a person's foot. Of course, everyone has a different sized foot so if we want to use, say the king's foot, as a standard, we will have to mark it permanently; we cannot very well lug the king around! Since the king is the ruler of the land, it was both rational and proper to call this marker a ruler too. To show the children the chaos caused by not having a standard length, have them measure the length of their desk in cubits or the width of the room in feet where each child uses his or her own body measurements. You will have as many different values as measurers.

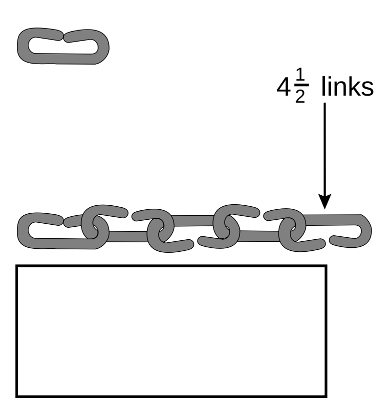
3.2 The Link System

At first we have the children measure the length of objects in links. A link is shown in Figure 1 along with a chain of links. The units of measurement of the object shown is 4½ links. The reason we go to links is (1) to make counting the unit length easy and (2) to keep the numbers manageable. Rather than dealing with hundreds of cm we can count 50 links. And to count a link is easy since they are so large.

By alternating link colors—say, 2 reds, 2 whites, 2 reds, etc., one can make the counting even easier by skip counting by 2's. If you want to have the children skip count by 5's, then have a chain with 5 blues, 5 yellows, 5 blues, etc.

With links it is also easy to round off to ½ link as shown in Figure 1. If the edge we are measuring is sort of near the middle, then the length is 4½ links. If shorter, then L = 4 links, if longer L = 5 links. This is all discussed in greater detail in *Rolling Along in Links.*

Figure 1



3.3 The Metric System

Of course, links is not a standard of measure. In this country, it is feet. Unfortunately, many objects are smaller than a foot. This means one must subdivide the standard unit. For some miserable reason, the smaller unit, the inch, is $1/12$ of a foot. This is LOUSY. Who likes to divide by 12! So, the French scientific community at the time of the French revolution (c. 1790) chose as the standard unit of length a distance which they called a meter. It was chosen so that 10^7 (10 million) metersticks laid end to end would just fit between the North Pole and the equator, as shown in Figure 2. A platinum-iridium rod was constructed with two marks a meter apart and stored in a vault near Paris. Every meterstick, albeit indirectly, comes from this standard. Having defined the meter, the French were smart enough to define all subsequent subdivisions of the meter as integral powers of ten. The foot is divided into $1/12$'s and $1/48$'s and other equally horrible numbers; for the meter, the divisions are $1/10$'s, $1/100$'s, and $1/1000$'s. We shall now explore this point further.

The meter is divided into three other units; the millimeter, the centimeter, and the decimeter (although the latter is rarely used). Without being specific as to the size of each unit we can order them using the greater than (>) or less than (<) sign. Starting with the smallest we have:

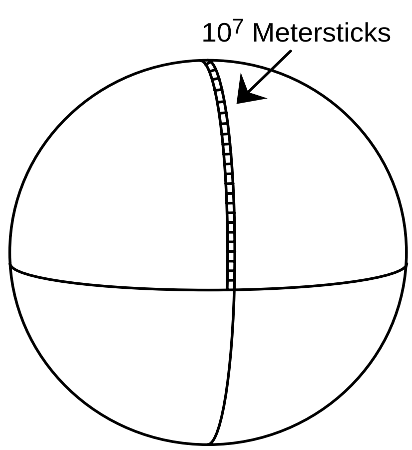
1 millimeter<1 centimeter<1 decimeter<1 meter.

Reversing the order and starting with largest, we have:

1 meter>1 decimeter>1 centimeter>1 millimeter.

It is important that the child know at least this much before going on to more exact relations.

Figure 2



The key to the subdivision of the meter is the word "milli." Milli is related to the word mile. Mile was the distance it took a Roman soldier to step off 1000 paces, a pace being two steps. Since an average pace is approximately five feet (try it and see), a mile would be approximately 5000 feet. The crucial point is the number 1000 as related to the word mile. Now milli is the prefix for "$1/1000$ of." Thus, a millimeter is one-thousandth of a meter. There are one thousand millimeters in a meter just as there are 1000 paces in a mile. In each case we have a subunit that is one-thousandth the main unit. The word mile is to remind us that there are 1000 paces in a mile. The word millimeter is to remind us that there are 1000 millimeters in a meter. One should try and picture this in one's mind. Millimeters are tiny; it takes a lot of them to make up any macroscopic length. (By macroscopic, we are referring to something one can see unaided versus microscopic where one would need a magnifying glass or microscope to see it.) On the other hand, because a meter is large, it is usually a fraction of most lengths one would measure in the lab. Fractions are hard to grasp and are usually not introduced until the 4th or 5th grade. Thus, it is reasonable that a 3rd grader knows how to measure the length of his thumb as 3 cm or 30 mm, but not as 0.03 m or $30/1000$ meters.

How are mm, cm, dm, and m related? Let's start with the smallest and see how many mm are in a cm, a dm, and a meter.

1 centimeter contains 10 mm;

1 decimeter contains 100 mm;

1 meter contains 1000 mm.

Based on these relationships, we should be able to figure out how many centimeters are in a decimeter, or in a meter. Here, however, the French have made it easy for us; the prefix for each word gives the answer away. “Centi” stands for 100th and “deci” stands for one-tenth. Thus one centimeter is one-hundredth of a meter; there are 100 cm in a meter. A decimeter is one-tenth of a meter; there are 10 dm in a meter. What this boils down to then is the following:

1 decimeter contains 10 cm;

1 meter contains 100 cm;

1 meter contains 10 dm.

Everything depends upon the size of the meter. Once that is fixed (by our rod in Paris), the sizes of all other metric units are determined.

3.4 Measuring Length

The simplest way to get started in the metric system is to count, using a meterstick, the number of mm or cm (and if they do not equal fractions, dm or meters) in a given length. Say we measure the width of a sheet of paper in cm. Then depending upon the accuracy of the meterstick (a cheap one could be off a bit) and the judgment of the student, one can see that there are between 21 and 22 cm across the page. If you stick to cm, then for 1st, 2nd, and 3rd graders this is all you can say since they do not know fractions or decimals. As they begin to learn decimals, we believe in the 4th and 5th grades, the children can determine the width as 21.6 cm. However, you can get still better accuracy even without decimals by going to mm instead of cm. Here, all one has to be able to do is count beyond 100. Thus the width is 216 mm. This is something that 3rd graders can do. In fact, one of the neat things about the metric system is that you can always choose a set of units to obtain almost any accuracy you want without going to fractions or decimals. On the other hand, for the upper grades, you can purposely choose units that will give decimal or fractional answers. As we just saw, in cm units, the width of the page is a decimal, 21.6 cm. We could have asked for the width in meters. Since the width is less than a meter, we are dealing with fractions. In this case, the width is 0.216 meters or roughly ⅕ of a meter. Clearly, there is great potential in the metric system for teaching math and linking this to scientific measurement. Math and science really go hand in hand and should be taught that way.

With regard to addition, when adding numbers they must always have the same units. Thus in the numerical sum only mm must appear or only meters. Never a mixture. For example:

(a) 5 cm + 6 m = ? This is a “no-no”; the units are mixed.

(b) 5 cm + 600 cm = 605 cm. This is okay—we are adding the same units.

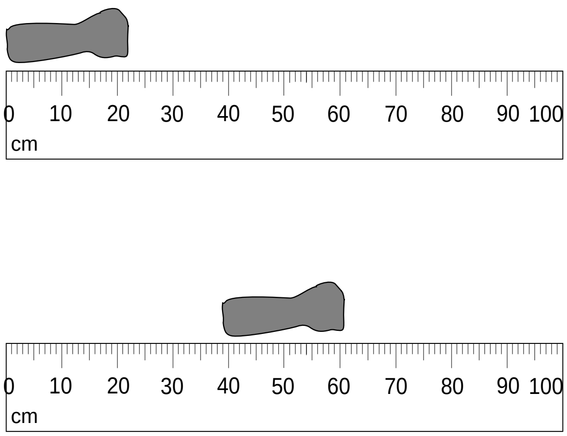
(c) 8 mm + 50 cm = ? We should convert the cm to mm and get

(d) 8 mm + 500 mm = 508 mm.

Thus, if we ask a student to measure the length of his arm by separately measuring his hand (say in mm), his lower arm (say in cm), his upper arm (say in decimeters), and then adding them, he will first have to convert to a set of consistent units that he can handle. For lower grade students, this would be mm since there will not be any fractions, while for upper graders the final answer might be in meters thus giving the children a chance to work on decimal fractions.

This brings us to another point, how to use a ruler. At first it seems quite apparent, just place the end of the ruler at the end of the object and read

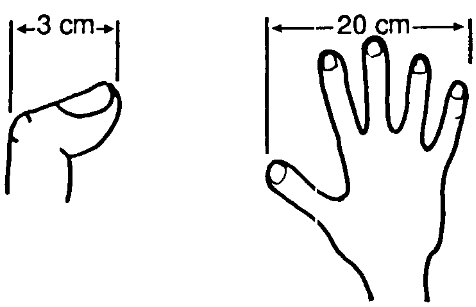
Figure 3



the length directly (Figure 3). However, a better test of whether students really understand how to use a ruler and also a test of their ability to subtract is to place the object in the center of the ruler. Clearly, the length of the object should not depend upon its position vis-a-vis the ruler, but we have found that many young people (and even a few at our university) have trouble understanding how to find the length in the latter case.

Since we do not always have a ruler handy, a few, shall we say "natural" rulers might be fun to discuss and use. For example, the length of my upper thumb from knuckle to tip is 3 cm while my spread-out fingers span 20 cm, as shown in Figure 4. Either can now be used to measure the length of an object. Of course, there is your foot, a convenient measure for stepping off distances. Mine, without my shoes, is 23 cm or about 9 inches. I guess I do not have the royal blood. Anyway, you should have the children measure a few objects using natural rulers and have them compare their results with that of a meterstick.

Figure 4



3.5 Length Preview

We now preview some of the TIMS experiments that will use the metric system and different techniques to measure length.

In the first three length experiments the children use links. They determine how far different carts roll after being launched from one incline in *Rolling Along in Links*, measuring arm span and height in *High, Wide, and Handsome*, and measuring coordinates in the *Mr. O* series. In these experiments one of the two primary variables, usually the responding variable, is length.

In later experiments both the manipulated and responding variables are length variables, and the measurements are in cm. We measure how high a ball bounces in *The Bouncing Ball*, how far a cart rolls from different height inclines in *Downhill Racer*, how high a sphere can roll up a hill in *Rolling Spheres*, how much you can see through a toilet paper roll in *View Tube*, and how your arm span and height are related in, not surprisingly, *Arm Span vs. Height.*

We combine length and time in our experiments called *Plant Growth*, and then we study average height vs. age as an addendum to *Arm Span vs. Height.*

We get a bit fancier and learn how to handle the length variable when the line is curved when we carry out *Circumference vs. Diameter*, when we study the important concept of angle in *Know All the Angles*, and in an advanced view tube experiment called *Getting the Range of It*. Yes, believe it or not, the correct definition of an angle really involves two length variables, one straight and one curved!

An important use of length comes in studying coordinate systems. The star here, besides the meterstick, is Mr. O who was developed by Piaget and Karplus to help children locate objects outside of themselves. We shall see how length helps us to determine where objects are, how to use signed numbers, how to make and read maps, and how to go from cm and meters to even blocks and miles as the basic units.

We have many other experiments where we combine length with one of our more advanced variables like area or volume. We shall turn to these later. But first and most importantly we learn about length.