Math Facts

TIMS Tutor

SECTION 9

Students need to learn the math facts. Estimation, mental arithmetic, checking the reasonableness of results, and paper-and-pencil calculations require the ability to give quick, accurate responses when using basic facts. The question is not *if* students should learn the math facts, but *how.* Which teaching methods are most efficient and effective? To answer this question, the authors of *Math Trailblazers* drew upon educational research and their own classroom experiences to develop a comprehensive plan for teaching the math facts. This tutor describes the teaching and learning of the math facts in the curriculum using the following outline:

Outline

This tutor is organized as follows:

Philosophy
Expectations by Grade Level
Strategies for Learning the Facts
- Addition
- Subtraction
- Multiplication
- Division

Math Fact Lessons
Practice
Assessment
Conclusion
References

Philosophy

The goal of the math facts strand in *Math Trailblazers* is for students to learn the basic facts efficiently, gain facility with their use, and retain that facility over time. A large body of research supports an approach which is built on a foundation of work with strategies and concepts. This not only leads to more effective learning and better retention, it also leads to development of mental math skills which will be useful throughout life. Therefore, the teaching and assessment of the basic facts in *Math Trailblazers* is characterized by the following elements:

- **Emphasis on problem solving.** Students learn the basic facts easily and naturally if they are encouraged to use a problem-solving approach to find answers to unknown facts. They first approach the math facts as problems to be solved rather than facts to be memorized. Many of the same thinking processes students develop to derive facts quickly are also useful for mental calculations and estimation. They can invent their own strategies or learn strategies through class discussions. If students are encouraged to use strategies and to share their thinking, they will continue to find math meaningful and related to what they already know.
- **Use of strategies.** Students should feel confident that they can think problems through to find answers they do not recall immediately. Therefore, we encourage the use of strategies to find facts and de-emphasize rote memorization.
- **Gradual introduction of the facts.** Students study small groups of facts which can be found using similar strategies. They first work on simple strategies for easy facts and practice these facts. Then, they can learn

more sophisticated strategies to learn harder facts and more efficient strategies to gain facility with all the facts.

- **Ongoing practice.** Work on the math facts is distributed throughout the curriculum. Students find a need to learn the facts as they encounter them in the labs, activities, and games. Systematic practice of small groups of facts is provided in the *Daily Practice and Problems.* Students are also encouraged to practice groups of facts at home on a regular basis.
- **Appropriate assessment.** Students are assessed through teacher observation as well as through the appropriate use of written tests and quizzes. Beginning in third grade, periodic short quizzes naturally follow the study of small groups of facts organized around specific strategies. These short quizzes are less threatening and as effective as longer tests, so we strongly recommend against the use of weekly testing of 60 to 100 facts. As self-assessment in third, fourth, and fifth grades, each student can record his or her progress on Facts I Know charts and determine which facts he or she needs to study. The goal of the math facts assessment program is to determine the degree to which students can find answers to fact problems quickly and accurately and whether they can retain this skill over time.
- **Facts will not act as gatekeepers.** Students are not prevented from learning more complex mathematics because they cannot perform well on fact tests. Use of strategies, calculators, and printed multiplication tables allow students to continue to develop number sense and work on interesting problems and experiments while they are learning the facts.

Expectations by Grade Level

Since we are committed to increasing and diversifying the mathematical content in the curriculum, our treatment of the basic facts differs from that in traditional textbooks. Specifically, we have the following goals:

In kindergarten, students use manipulatives and invent their own strategies to solve addition and subtraction problems.

By the end of first grade, all students can solve all basic addition and subtraction problems using some strategy. Facility is not emphasized; strategies are.

In second grade, strategies for addition and especially subtraction continue to be emphasized. Some work with beginning concepts of multiplication takes place. By the end of the year, all students are expected to demonstrate facility with all the addition facts.

In third grade, strategies for subtraction are encouraged, leading to facility with all the subtraction facts by the end of the year. Development of the concept of division and strategies for the multiplication facts are included.

In fourth grade, we develop strategies for the multiplication and division facts. By the end of year, we expect facility with all the multiplication facts.

By the end of fifth grade, students are expected to demonstrate facility with all the division facts as well as the addition, subtraction, and multiplication facts.

These benchmarks are somewhat later than is traditional for a number of compelling reasons. First, concepts and skills are learned more easily and are retained better if they are meaningful. By first concentrating on concepts and strategies, we increase retention and reduce the amount of time necessary for rote memorization. Second, we spread the work on the facts over more years so that more time is made available at all grade levels for other topics such as measurement, probability, and statistics that have traditionally been neglected.

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Studying more rigorous content provides practice using the facts in meaningful settings and furnishes intrinsic motivation for gaining facility with the facts. Third, the de-emphasis of paper-and-pencil computation in the elementary curriculum has reduced the need for early mastery of the facts. If long division, multidigit multiplication, and paper-and-pencil procedures in general receive less emphasis, then students do not need to learn the facts as early, even though their eventual importance is undiminished.

Grade	Addition	Subtraction	Multiplication	Division
K	• invented strategies	• invented strategies		
1	• strategies	• strategies		
2	• strategies • practice leading to facility	• strategies		
3	• review and practice	• strategies • practice leading to facility	• strategies	• strategies
4	• assessment and remediation as required	• assessment and remediation as required	• strategies • practice leading to facility	• strategies
5			• review and practice	• strategies • practice leading to facility

Table 1: Math Facts Scope and Sequence

Strategies for Learning the Facts

As stated in the previous sections, students are encouraged to learn the math facts by first employing a variety of strategies. Over time, students develop techniques that are increasingly sophisticated and efficient. In this section, we describe possible strategies for learning the addition, subtraction, multiplication, and division facts. The strategies for each operation are listed roughly in order of increasing sophistication.

Strategies for Addition Facts

Common strategies include counting all, counting on, doubles, making or using 10, and reasoning from known facts.

Counting all

This is a particularly straightforward strategy: to solve 7 + 8, for example, the student gets 7 of something and 8 of something and counts how many there are altogether. The “something” could be beans or chips, or marks on paper. In any case, the student counts all the objects to find the sum. This is perhaps not a very efficient method, but it is effective, especially for small numbers, and is usually well understood by the student.

Counting on

This is a natural strategy, particularly for adding 1, 2, or 3. Counters such as beans or chips may or may not be used. As an example with counters, consider 8 + 3. The student gets 8 beans, and then 3 more, but instead of counting the first 8 again, she simply counts the 3 "new" beans: "9, 10, 11."

Even if counters are not used, finger gestures can help keep track of how many more have been counted on. For example, to solve 8 + 3, the student counts "9, 10, 11," holding up a finger each time a number word is said; when three fingers are up, the last word said is the answer. There are efficient finger techniques you can teach your students if you wish, or you can let them use their own intuitive methods.

Doubles

Facts such as 4 + 4 = 8 are easier to remember than facts with two different addends. Some visual imagery can help, too: two hands for 5 + 5, a carton of eggs for 6 + 6, a calendar for 7 + 7, and so on.

Making a 10

Facts with sum 10, such as 7 + 3 and 6 + 4, are also easier to remember than other facts. A ten frame can be used to develop imagery to help even more. For example, 8 is shown in a ten frame like the one in Figure 1:

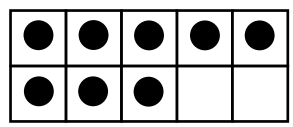


Figure 1: A ten frame

This sort of visual imagery helps the students remember, for example, that 8 + 2 = 10.

Using a 10

Students who are comfortable partitioning and combining small numbers can use that knowledge to find the sums of larger numbers. In particular, there are many strategies that involve using the number 10. For example, to find 9 + 7, we can decompose 7 into 1 + 6 and then 9 + 7 = 9 + 1 + 6 = 10 + 6 = 16. Similarly, 8 + 7 = 8 + 2 + 5 = 10 + 5 = 15.

Reasoning from known facts

If you know what 7 + 7 is, then 7 + 8 is not much harder: it's just 1 more. So, the "near doubles" can be derived from knowing the doubles. This is an example of reasoning from known facts.

Strategies for Subtraction Facts

Common strategies for subtraction include using counters, counting up, counting back, using 10, and reasoning from related addition and subtraction facts.

Using counters

This method consists of "acting out" the problem with counters like beans or chips. For example, to solve 8 – 3, the student gets 8 beans, removes 3 beans, and counts the remaining beans to find the difference. As with the addition strategy of "counting all," this is a relatively straightforward strategy that may not be efficient but has the great advantage of usually being well understood by the student.

Counting up

The student starts at the lower number and counts on to the higher number, perhaps using fingers to keep track of how many numbers are counted. For example, to solve 8 – 5, the student wants to know how to get from 5 to 8 and counts up 3 numbers: 6, 7, 8. So, 8 – 5 = 3. Special finger-counting techniques may be helpful, but this strategy seems to be a natural for most students.

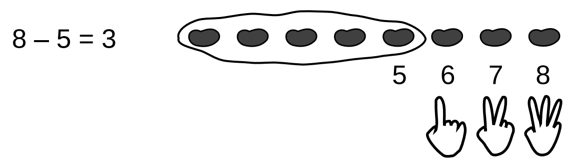


Figure 2: Counting up

Counting back

Counting back works best for subtracting 1, 2, or 3; for larger numbers, it is probably best to count up. For example, to solve 9 – 2, the student counts back 2 numbers: 8, 7. So, 9 – 2 = 7.

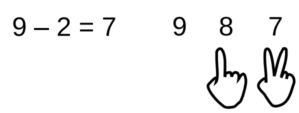


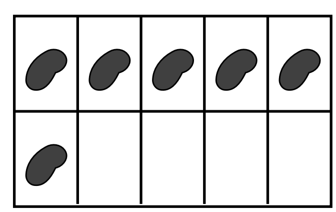
Figure 3: Counting back

Using a 10

Students follow the pattern they find when subtracting 10, e.g., 17 – 10 = 7 and 13 – 10 = 3, to learn "close facts," e.g., 17 – 9 = 8 and 13 – 9 = 4. Since 17 – 9 will be 1 more than 17 – 10, they can reason that the answer will be 8, or 7 + 1. In this strategy, 10 is a part in the whole-part-part scenario.

Making a 10

Knowing the addition facts which have a sum of 10, e.g., 6 + 4 = 10, can be helpful in finding differences from 10, e.g., 10 – 6 = 4 and 10 – 4 = 6. Students can use ten frames to visualize these problems as shown in Figure 4. These facts can then also be used to find close facts, such as 11 – 4 = 7. In this strategy, 10 is the whole in whole-part-part scenario.



10 – 4 = 6

Figure 4: Using a ten frame

Using doubles

The addition doubles, e.g., 8 + 8 = 16 and 6 + 6 = 12, can be used to learn the subtraction "half-doubles" as well: 16 – 8 = 8 and 12 – 6 = 6. These facts can then be used to figure out close facts, such as 13 – 6 = 7 and 15 – 8 = 7.

Reasoning from related addition and subtraction facts

Knowing that 8 + 7 = 15 would seem to be of some help in solving 15 – 7. Unfortunately, however, knowing related addition facts may not be so helpful to younger or less mathematically mature students. Nevertheless, reasoning from known facts is a powerful strategy for those who can apply it and should be encouraged.

Strategies for Multiplication Facts

Common strategies for multiplication include skip counting, counting up or down from a known fact, doubling, breaking a product into the sum of known products, and using patterns.

Skip counting

Students begin skip counting and solving problems informally that involve multiplicative situations in first grade. By the time formal work with the multiplication facts is begun in third grade, they should be fairly proficient with skip counting. This strategy is particularly useful for facts such as the 2s, 3s, 5s, and 10s for which skip counting is easy.

Counting up or down from a known fact

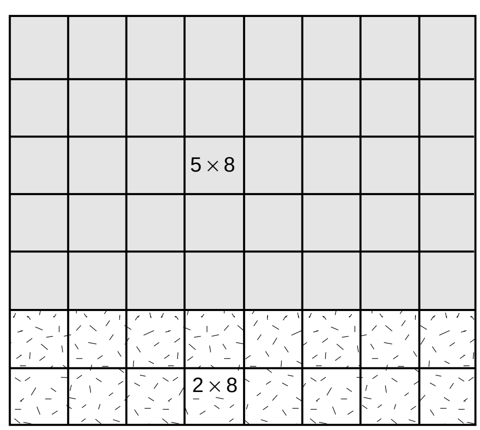
This strategy involves skip counting forwards once or twice from a known fact. For example, if a child knows that 5×5 is 25, then this can be used to solve 6×5 (5 more) or 4×5 (5 less). Some children use this for harder facts. For 7×6, they can use the fact that $5 \times 6 = 30$ as a starting point and then count on by sixes to 42.

Doubling

Some children use doubling relationships to help them with multiplication facts involving 4, 6, and 8. For example, 4×7 is twice as much as 2×7. Since $2 \times 7 = 14$, it follows that 4×7 is 28. Since 3×8 is 24, it follows that 6×8 is 48.

Breaking a product into the sum of known products

A fact like 7×8 can be broken into the sum $5 \times 8 + 2 \times 8$ since $7 = 5 + 2$. (See Figure 5.) The previous two strategies are special cases of this more general strategy.



$$7 \times 8 =$$
$$5 \times 8 + 2 \times 8 =$$
$$40 + 16 = 56$$

Figure 5: Breaking up 7×8

Patterns

A. Perhaps the best-known examples of patterns are the nines patterns:

1. When the nines products are listed in a column, as shown below, it is easy to see that the digits in the tens place count up by one (0, 1, 2, 3, ...) and that the digits in the ones place count down by one (9, 8, 7, ...).

    9
    18
    27
    36
    45
    54
    63
    72
    81

2. The sums of the two digits in each of the nines products above are all equal to nine. For example, the sum of the digits in 36 is 3 + 6 = 9; the sum of the digits in 72 is 7 + 2 = 9. Adding the digits of a number to see whether they add up to nine can be a strategy in remembering a nines fact. For example, a student might think, "Let me see, does 9×6 equal 54 or 56? It must be 54 since 5 + 4 is 9, but 5 + 6 is not 9."

3. The digit in the tens place in a nines fact is one less than the number being multiplied. For example, $4 \times 9 = 36$, and 3 is one less than 4. This can be combined with the previous pattern to derive nines facts. For example, 3×9 is in the twenties. Since 2 + 7 is 9, 3×9 must be 27.

4. Nines can easily be computed using the counting down strategy. Nine times a digit is the same as 10 times the digit, minus the digit. For example, 9×6 is $10 \times 6 - 6 = 54$. This works well because multiplying by 10 is so easy.

B. Other patterns: There are other patterns which can be useful in remembering other special facts:

1. 0 times a number equals 0.
2. 1 times a number equals the number.
3. 2 times a number is double the number.
4. 5 times a number ends in 0 or 5; even numbers times 5 end in 0 and odd numbers times five end in 5.
5. 10 times a number is the same number with a 0 on the end.

Sequencing the Study of Multiplication Facts

In kindergarten, children solve word problems involving multiplication situations. Beginning in first grade, the curriculum develops a conceptual foundation for multiplication through a variety of multiplication models, including the set model, array model, and number line model. Introduction of strategies to gain facility with the multiplication facts begins in Unit 13, *Multiplication Patterns,* of third grade. We do not introduce the multiplication facts in the order in which they are traditionally taught (first learning the 2s, then the 3s, then the 4s, etc.). Rather, we emphasize thinking strategies for the facts in the following order:

0s, 1s, 2s, 3s, 5s, and 10s. We call these the "Handy Facts." The 2s, 3s, 5s, and 10s are easily solved using skip counting (Third grade, Unit 13, Lesson 2).

Square numbers such as $3 \times 3 = 9$, $4 \times 4 = 16$, and $5 \times 5 = 25$. These are introduced by arranging tiles into square arrays (Third grade, Unit 13, Lesson 3).

Nines. Students explore patterns for nines in third grade. (Unit 13, Lesson 4)

Last six facts. After students have learned the facts listed above and their "turn around facts" ($9 \times 6 = 6 \times 9$), there are only six more facts to learn: 4×6, 4×7, 4×8, 6×7, 6×8, and 7×8.

Strategies for the Division Facts

The main strategy for learning the division facts is to think of the related multiplication fact. Therefore, mastery of division facts should be delayed until children have reasonable facility with the multiplication facts.

Using the "Right" Strategy

It is important to remember that different strategies appeal to different students. Students should not feel overburdened with the need to determine which is the "correct" strategy for a given fact. We do not intend to give them a new layer of things to learn. For example, when asked to explain a strategy for a fact, a student may say, "I've used it so much that now I just remember it." "Just remembering" is obviously an efficient strategy. The purpose of suggesting and discussing various strategies is to give students other, perhaps helpful, ways of learning the facts and to give them the confidence to think problems through when necessary. Students should have the opportunity to choose the strategies which work best for them or to invent their own.

Math Facts Lessons

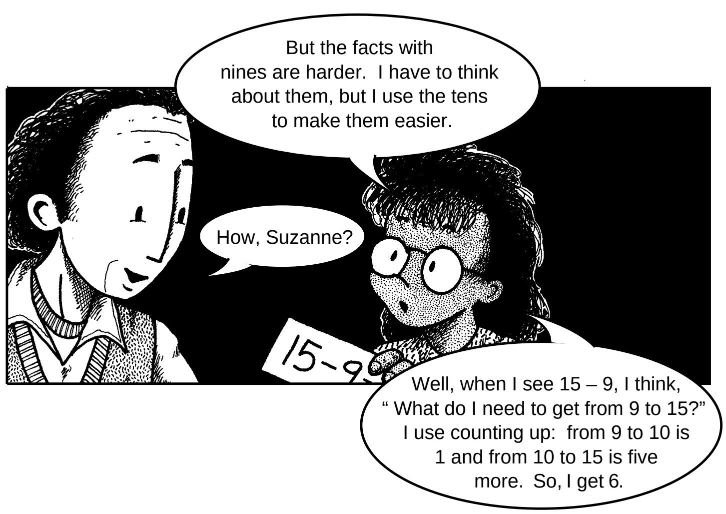


Figure 6: Discussing fact strategies

Encouraging the everyday use of strategies such as those above is the most important part of our approach to the basic facts. Occasionally, however, we recommend special lessons that focus on particular strategies and explore how and why they work. Since our objective here is facility with all the facts, students are encouraged to choose strategies which they find effective in learning the facts.

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Everyday work. As students work on problems in the labs and activities, they should be encouraged to use and to discuss various strategies for solving "math facts" problems. A number of important goals can best be reached through such discussions.

One goal is to legitimize all valid strategies, even those that may be "less efficient." When students see their intuitive methods recognized and validated, they tend to perceive mathematical knowledge as continuous with everyday knowledge and common sense. We thus hope to avoid the unfortunate tendency of many students to separate their knowledge of mathematics from their knowledge of the real world.

By discussing strategies as they arise in context, students and the teacher can explore how the strategies work and can verify that they are being used properly. Students should come to realize that a fast strategy that gives wrong answers is not very useful.

A second goal of our approach is to get the children used to communicating mathematics. There are several reasons to stress communication: Students can learn from one another; communicating a method requires higher orders of thinking than simply applying that method; and skill at communicating is important in itself. We are social creatures. Mathematics and science are social endeavors in which communication is crucial.

A third goal of encouraging discussions of various methods is to give the teacher opportunities to learn about how students think. Knowing more about students' thinking helps the teacher ask better questions and plan more effective lessons.

A final goal of discussing many strategies for solving problems is to emphasize that methods are often as important as answers. Too many students focus excessively on filling papers with correct answers. To paraphrase Thoreau, give a student an answer and you have fed her for a day; teach her a method and you have fed her for life.

Strategy lessons. We feel that occasionally it is appropriate for lessons to focus on certain strategies that are developmentally appropriate for most of your students. Our plan is to begin with simple strategies that should be accessible to all students and to progress gradually to more complex forms of reasoning. For example, in the fall of first grade, we have several lessons that stress counting on to solve certain addition problems. Later, we explicitly introduce making a 10 and other, more sophisticated, strategies.

In general, you should expect your students to come up with effective strategies on their own. Our "strategy lessons" are intended to explore how and why various strategies work and also to codify and organize the strategies the students invent. They are not meant to dictate the only appropriate strategy for a given problem or to discourage students from using strategies they understand and like. They should be seen as opportunities to discuss strategies that may be appropriate for many students and to encourage their wider use.

Practice

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Our ultimate goal is to produce students who can think mathematically, who can solve problems and deal easily with quantified information, and who enjoy mathematics and are not afraid of it. It is easier to do all of the above if one has facility with the basic math facts. We recommend, and have incorporated into the curriculum, the following practice to gain this facility.

Practice in Context

The primary practice of math facts will arise naturally for the students as they participate in the labs and other activities in the curriculum. These labs and activities offer many opportunities to practice addition, subtraction, multiplication, and division in a meaningful way. Our math facts lessons involve the student visually with drawings and patterns, auditorily through discussion, and tactually through the use of many tools such as manipulatives and calculators.

Pages of problems on the basic facts are not only unnecessary, they can be counterproductive. Students may come to regard mathematics as mostly memorization and may perceive it as meaningless and unconnected to their everyday lives.

Structured Practice

Student-friendly structured practice is built into the curriculum, especially in the *Daily Practice and Problems* and games. One small group of related math facts is presented to the students at a time. The practice of groups of facts is carefully distributed throughout the year. A small set of facts grouped in a meaningful way leads students to develop strategies such as *adding doubles, counting back,* or *using a 10* for dealing with that particular situation. Furthermore, a small set of facts is a manageable amount to learn and remember. Beginning in third grade, the learning of each small set of facts is assessed separately. Students are less stressed and more successful on the quizzes. They come to feel confident rather than overwhelmed about their abilities to remember the facts.

The *Daily Practice and Problems* is the place where much of the structured fact practice takes place. Beginning in third grade, a small group of facts to be studied in a unit is introduced here. Flash cards are presented, practice on the facts is given, and a quiz is provided. Facts are also practiced in many other problems of the DPP where the emphasis is on other interesting math. These problems allow students to focus on other interesting mathematical ideas as they are also getting more fact practice.

Games

A variety of games are included in the curriculum, both in the lessons and in the *Daily Practice and Problems* that are included in each unit. The games list can be found in the *Teacher Implementation Guide.* Once students learn the rules of the games, they should play them periodically, both in class and at home for homework. Games provide an opportunity to encourage family involvement in the math program. When a game is assigned for homework, a note can be sent home with a place for the family members to sign, affirming that they played the game with their student.

Figure 7: Playing a game

Use of Calculators

The relationship between knowing the math facts and the use of calculators is an interesting one. Using a multiplication table or a calculator when necessary to find a fact helps promote familiarity and reinforces the math facts. Students soon figure out that it is quicker and more efficient to know the basic facts than to have to use these tools. The use of calculators also requires excellent estimation skills so that one can easily check for errors in calculator computations. Rather than eliminating the need for facility with the facts, successful calculator use for solving complex problems depends on fact knowledge.

When to Practice

Practicing small groups of facts often (for short periods of time) is more effective than practicing many facts less often (for long periods of time). For example, practicing 8 to 10 subtraction facts for 5 minutes several times a week is better than practicing all the subtraction facts for half an hour once a week. Good times for practicing the facts for 5 or 10 minutes during the school day include the beginning of the day, the beginning of math class, when students have completed an assignment, when an impending activity is delayed, or when an activity ends earlier than expected. Practicing small groups of facts at home involves parents in the process and frees class time for more interesting mathematics.

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Having a "math-literate" classroom will give your students many natural and meaningful chances to use mathematics and gain more practice with their math facts. We encourage you to look for opportunities to use mathematics with your students in the many occasions that arise in other subjects and throughout the day.

Assessment

The assessment of the math facts is closely aligned with the philosophy and organization of their instruction. Throughout the curriculum, teachers assess students' knowledge of the facts through observations as they work on activities, labs, and games. In grades 3–5, students can use their Facts I Know charts to record their own progress in learning the facts. This type of self-assessment is very important in helping each student to become responsible for his or her own learning. Students are able to personalize their study of facts and not waste valuable time studying facts they already know.

At the third-grade level and above, a sequence of tests and quizzes is provided in the *Daily Practice and Problems.* The tests and quizzes assess facility of the facts appropriate at each grade level as outlined in the Expectations by Grade Level section of this tutor. As students develop strategies for a given group of facts, short quizzes accompany the practice. Students know which facts will be tested, focus practice in class and at home on those facts, then take the quiz. As they take the quiz, they use one color pencil to write answers before a given time limit, then use another color to complete the problems they need more time to answer. With the results of this type of quiz, students can use their Facts I Know charts to make a record of those facts they answered quickly, those facts they answered correctly but with less efficient strategies, and those facts they did not know at all. Using this information, students can concentrate their efforts on gaining facility with those facts they answered correctly, but not quickly. They also know to develop strategies for those facts they could not answer at all. In this way, the number of facts studied at any one time becomes more manageable, practice becomes more meaningful, and the process less intimidating.

The practice as well as the assessment of groups of facts is carefully distributed throughout the year. For example, in third grade, 72 subtraction facts are divided into eight groups of nine facts each. Each group is organized around a strategy such as *counting back* or *using a 10.* The fact groups are introduced and practiced (two groups at a time) in lessons and the *Daily Practice and Problems* for Units 2–5. These fact groups are reviewed, practiced, and then assessed through a series of quizzes in the DPP in Units 6–10. Unit 10 also includes a test of all 72 subtraction facts. The process is repeated in the second semester (Units 11–20) where each group of facts is reviewed and practiced two more times as part of the *Daily Practice and Problems* and assessed in a second series of quizzes. A second test of all 72 facts is given in the final unit.

The distribution of fact practice and assessment is similar in fourth and fifth grades. Such ongoing assessment is consistent with the goals of the math facts program. Students are evaluated on their abilities to use basic number facts quickly and accurately and to retain their skills over time. Tests of all the facts for any operation have a very limited role. They are used no more than two or three times a year to show growth over time and should not be given daily or weekly. Since we rarely, if ever, need to recall 100 facts at one time in the real world, overemphasizing tests of all the facts reinforces the notion that math is nothing more than rote memorization and has no connection to the real world. Quizzes of small numbers of facts are as effective and not as threatening. They give students, parents, and teachers the information needed to continue learning and practicing efficiently. With an assessment approach based on strategies and the use of small groups of facts, students can see mathematics as connected to their own thinking and gain confidence in their mathematical abilities.

Conclusion

Students enter school with considerable informal knowledge of mathematics. A challenge for teachers is to build on that knowledge, to make school mathematics continuous with what the students already know. One effective way to help students see connections between school mathematics and their lives outside of school is to encourage them to use their own informal strategies for solving simple addition and subtraction problems.

Recent research indicates that students learn the addition and subtraction facts easily and naturally if they are encouraged to use their own strategies. There are also indications of increased mathematical confidence in students who are encouraged to think problems through rather than rely on rote memorization. They will learn their facts as well if not better than with lots of drill, but more importantly, they will learn that mathematics is meaningful and that they can figure things out for themselves.

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