

NAME :

**Open book, open notes, open computer, but closed mouth!**  
**Write all answers on these sheets.**

question	1	2	3	4	5	6	7	total
points								
maximum	15	10	20	10	15	20	10	100

1. Define in Maple the function  $f(x) = 2.3^{-x^2}$ .

(a) Give all commands to create a function  $t$  which approximates  $f$  using a 10-th order Taylor series for  $f$  about  $x = 1$ .

(b) What is the difference between  $t(1.2)$  and  $f(1.2)$ ?

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2. Do  $X := [\text{seq}(x[i], i=1..100)]$ ; and give **one** command to decimate the list, i.e.: remove every tenth element of  $X$ .

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3. Denote the composite Trapezoidal rule for  $\int_a^b f(x)dx$  using  $2^n$  intervals by  $T[n](f, a, b)$ .

We can define  $T[n](f, a, b)$  recursively by two rules:

$$T[0](f, a, b) = (f(a) + f(b)) * (b-a) / 2;$$

$$T[n](f, a, b) = T[n-1](f, a, (a+b)/2) + T[n-1](f, (a+b)/2, b), \text{ if } n > 0.$$

- (a) Write a recursive Maple procedure for  $T$ , where  $n$  must be an index to  $T$ .

- (b) Explain what *the user* of  $T$  must do to prevent that  $f$  is never evaluated twice at the same point.

Illustrate using  $n = 5$  in  $T$  for the numerical approximation of  $\int_0^1 \cos(x)dx$ .

4. Explain the difference between `diff` and `D`.

Give a good example of both commands to illustrate their use.

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5. The curve defined by  $2x^4 - 3x^2y + y^2 - 2y^3 + y^4 = 0$  has a *tacnode* at  $(0, 0)$ .  
Give the Maple commands (not the output) to

(a) plot this curve for  $(x, y) \in [-2, +2] \times [0, 2.5]$ :

(b) transform the equation into polar coordinates:

(c) make a plot using polar coordinates. Ensure your plot is complete! (see (a))

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6. Consider the system 
$$\begin{cases} x^3 + x^2y - x^2 - 2x - 2y + 2 = 0 \\ 2x^2 + 3xy - 2y^2 = 0. \end{cases}$$

(a) Give the Maple command(s) to bring this system in triangular form.

(b) How many solutions are there? How many are rational? Explain!

(c) Give the commands to compute all solutions, and to create a list of their coordinates in the form  $[\mathbf{x}, \mathbf{y}]$ .

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7. Use an index function to define the multiplication table modulo 7.  
This multiplication table is a 6-by-6 matrix whose  $(i, j)$ th entry is  $i \times j \pmod{7}$ .

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