

Mathematic 300, Spring 2003

In Class Writing
Convergence

In the various readings and discussions for this class, one can find the following four statements.

1. Euler proved $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$.
2. $\sum_1^\infty \frac{1}{2^n} = 2$.
3. $\sum_1^\infty \frac{1}{n^2} < 1.7$.
4. $\sum_1^\infty \frac{1}{n^3}$ is between 1.202 and 1.203.

Is there any difference ‘in kind’ between statements 1 and 2? Is there any difference ‘in kind’ between statements 3 and 4? Or do they just make similar statements about different series? Does this mean $\sum_1^\infty \frac{1}{n^3}$ has no exact value? What technique (no computation; describe the method) would you use to establish the third and fourth statements. With this as background, explain the meaning of the assertion that the problem of computing $\sum_1^\infty \frac{1}{n^3}$ is open.

(Details can be found on the web site

<http://www.mat.bham.ac.uk/C.J.Sangwin/Teaching/pus/infersup.pdf>)