Math 300 Writing for Mathematics Spring 2003

Short Writing Exercise 1

Part I. Rewrite the passage given below by replacing *at least* three words in each sentence with words that mean approximately the same thing. Be sure to reread what you write before turning in the assignment. Check that the spelling and punctuation are correct and the sense of the passage remains the same.

This assignment is a taken from D. E. Knuth, T. Larabee, and P. M. Roberts, *Mathematical Writing*, MAA Pub., 1989. The passage is taken from pages 479-480 of What is Mathematics: An Elementary Approach to Ideas and Methods, Courant and Robbins, Oxford University Press, first paperback edition 1978. You may find this book a useful source for this course as well as a great introduction to mathematics.

Part II. Now rewrite the section beginning, 'Although the terms . . . ', in your own words. You may want to group the terms in slightly different ways (compare the essay: the Mathematics of Essay 1 on the website). The mathematics for this discussion is going to be about the same in any book or paper. The context and the detailed explanation should show that you understand the result. In Essay 1 you will have to connect this formal treatment with the blocks. You may find it useful to use (a modification of) your answer to this question as part of your first essay.

> Series whose terms are simple combinations of the integers are particulary interesting. As an example we consider the 'harmonic series'":
>
> $$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\cdots+\frac{1}{16}+\cdots, \qquad (1)$$
>
> which differs for that for $\ln 2$ by the signs of the even-numbered terms only. To ask whether this series converges is to ask whether the sequence
>
> $$s_1, s_2, s_3, \cdots, \qquad (2)$$
>
> where
>
> $$s_n = 1+\frac{1}{2}+\frac{1}{3}+\cdots\frac{1}{n} \qquad (3)$$
>
> tends to a finite limit. Although the terms of the series 1 approach 0 as we go out farther and farther, it is easy to see that the sequence does not converge. For by taking enough terms we can exceed any positive number whatsoever, so that s_n increases without limit and hence the series 1 'diverges to infinity'. To see this we observe that
>
> $$s_2 = 1+\frac{1}{2},$$
>
> $$s_4 = s_2 + (\frac{1}{3}+\frac{1}{4}) > s_2 + (\frac{1}{4}+\frac{1}{4}) = 1+\frac{2}{2},$$

$$s_8 = s_4 + (\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}) > s_4 + (\frac{1}{8} + \cdots \frac{1}{8}) = s_4 + \frac{1}{2} > 1 + \frac{3}{2},$$

and in general

$$s_{2^m} > 1 + \frac{m}{2}.$$

Thus, for example, the partial sums s_{2^m} exceed 100 as soon as $m \geq 200$.