Similar triangles, incenter and proportionality.*

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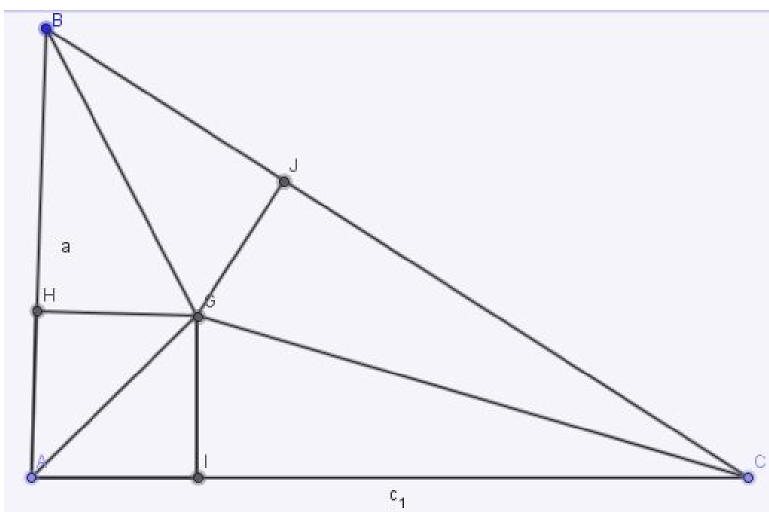
1) A park is surrounded by three straight lines AB,BC, CA. The park authority wants to put a statue at a point O in the park that is equidistant from all three lines. How can one construct O?

2) Draw lines from O to points D, E, F on the three sides of the triangle so that the lines are perpendicular to the side of the triangle. How do you prove that the three lines are equal in length.

3) Does this construction minimize $OD + OE + OF$? (See `http://www.gogeometry.com/center/triangle_center_fermat_point_equilateral.htm`.)

4) Can you find word problems that make this construction more enticing?



Definition 0.1 *Two triangle are* similar *if there is a correspondence between them making corresponding angles equal.*

5) Prove that ABC and $A'B'C'$ are similar then **using the segment multiplication we have defined in class**

*UIC CTTI: Dec. 3, 2012

$$\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'}.$$

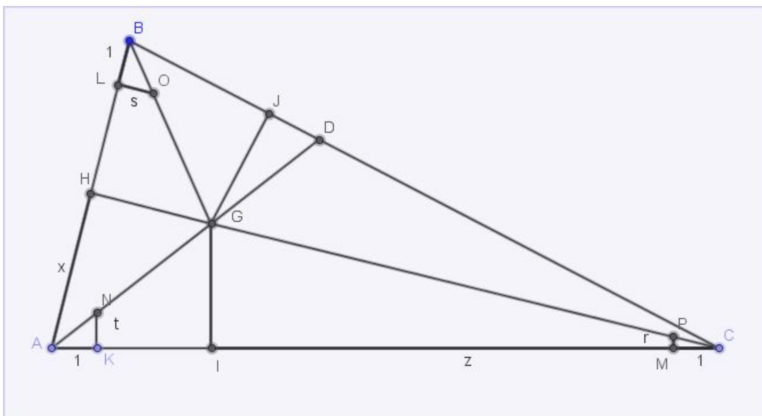
HINT Use the construction of part 1).

Consider the triangle ABC below. Note that by the construction above $HG \cong GI \cong GJ$. Call this segment length a.

Now construct $AK \cong BL \cong MC$ all with segment length our standard 1. Let the lengths of BL be s, NK be t and PM be r.

Let the lengths of $AI \cong AH$ be y, $BH \cong BJ$ be x, and $CI \cong AJ$ be z.

By the definition of multiplication $t \cdot y = r \cdot z = a$. Therefore the length of AC is $\frac{a}{t} + \frac{a}{r} = \frac{a(r+t)}{rt}$.



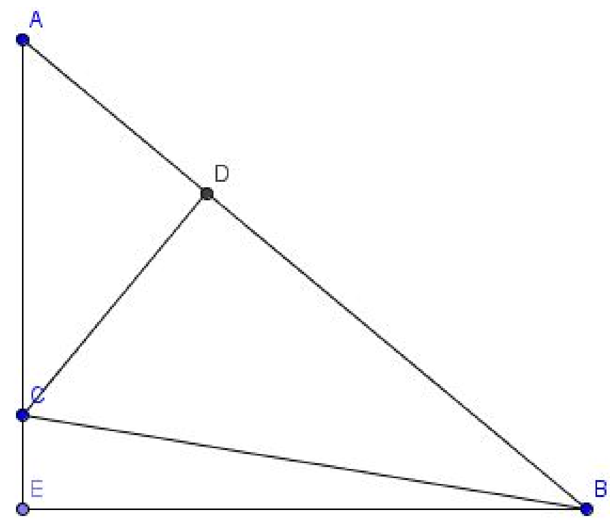
Duplicate on second triangle $A'B'C'$ to get the the length of $A'C'$ is $\frac{a'}{t} + \frac{a'}{r} = \frac{a'(r+t)}{rt}$. (The crucial point is that because the angles are congruent r, s, t are the same for both triangles.)

But then $\frac{A'C'}{AC} = \frac{a'}{a}$. Now note the same is true for the other two pairs of sides so the sides of the triangle are proportional.

6) Prove that all choices of altitude and base of a triangle give the same value for $\frac{bh}{2}$.

Consider the triangle ABC below. Two altitudes are DC and EB. Note that the triangle EAB and DAC are similar. Apply problem 5.

Make sure you check the third altitude.


A
D
C
E
B