Supplement to First Midterm: Math 215

February 24, 2007

Name:

If you missed items 3, 4, or 5 on the midterm, you should do the problem with the same number below. I will grade papers that are signed by two people indicating that you have convinced each other that the solutions are correct.

3 Prove from the axioms on ordered fields that

1. There is no greatest real number a such that $a^2 < 2$.
2. There is no greatest rational number a such that $a^2 < 2$.
3. Explain the difference between the two results.

4 You may assume basic algebraic properties without explicit reference to the axioms in this problem. Recall that for integers d, s, n, 'd divides n means there is an s such that $n = ds$. Let d, x and y be integers. Prove

1. If d divides x and d divides y then d divides $x + y$.
2. If d divides $x + y$ and d divides y then d divides x.
3. Explain the connection between the two results.

5 Consider the following three statements:

1. If $\frac{6x+5}{x+2} < 5$, then $x < 5$.
2. $x < 5$ then $\frac{6x+5}{x+2} < 5$.
3. $x < 5$ if and only if $\frac{6x+5}{x+2} < 5$.

Which of them are true? Which one of them is needed to prove by induction that if $a_{k+1} = \frac{6a_k+5}{a_k+2}$ then for all n, $a_n < 5$?