Study Guide for Final exam Math 215

The exam will cover the book up to the chapter with the divison algorithm (15?).

Here are some sample questions.

1. Write down the negation of the statement: $(\forall x \in \mathbb{Z})(\forall w \in \mathbb{Z})(\exists y \in \mathbb{Z})(xy < w)$. Is it true?
2. Define the following terms
   1. X and Y are equipotent.
   2. $(X, <)$ is a linearly ordered set
   3. $f : X \mapsto Y$ is a surjection.
3. Let f map X to Y:
   1. What are the domain and codomain of $\overleftarrow{f}$
   2. What is the difference between the range of f and the codomain of f.
   3. What is the pigeonhole principle
4. Prove each of the following if it is true or give a counterexample.
   1. If $|X| = |Y|$ and f is an injection from X to Y then f is surjective. Does it make a difference if X is finite?
   2. If A is infinite $|\mathbb{N}| \leq |A|$.
   3. If A is countable then A is infinite.
   4. Every surjective function is injective.
5. Use calculus to show the function $f(x) = x^5$ is 1-1.
6. The dyadic rationals is the set $D = \{\frac{a}{2^n} : a \in \mathbb{Z}, n \in \mathbb{N}\}$. Show D is countable.
7. Show that if n is odd, 9 divides $8^n + 1$.
8. Sketch the proof that there are uncountably many real numbers.
9. Prove $A \cap B^c = \emptyset$ implies $A \subseteq B$.
10. Suppose $a < b$ and $c < d$ are real numbers and the interval (a, b) intersects the interval $[c, d]$. What can you say about the ordering of the 4 numbers.