# Math 300: Writing in Mathematics

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# LOGISTICS



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CLASS APR 1 OR MAR 30
office hours: 10:00 AM M; 11 AM T 11 AM W (tentative)
Assignments will be on the web.
There will be 3 essays (2 drafts of each and outline of 2nd two) and various short writing assignments. Some will be in class and some for homework.

# How not to write!

What does the following sentence from the Chicago Tribune on Jan. 8, 2004 mean?

Between 6 and 8 p.m. most days in November, up to 70 per cent of flights arrived late at O'Hare.

# References



Why do we give references?

# Some possible answers



Justify our statements.
Enable the reader to learn more.
Give credit to others.

# Plagiarism



What is plagiarism?

# Some possible answers

Google: a piece of writing that has been copied from someone else and is presented as being your own work

en.wikipedia.org/wiki/Plagiarism:
Plagiarism is the unauthorized use or close imitation of the language and thoughts of another author and the representation of them as one's own original work.

`http://www.google.com/search?hl=en&client=firefox-a&rls=org.mozilla:en-US:official&hs=XYb&defl=en&q=define:plagiarism&sa=X&oi=glossary_definition&ct=title`

# What is the interest rate?



A bank issues the following prospectus for a CD.

minimum investment: $10,000
interest rate: 3.92 %
Compounded daily:
annual percentage yield: 4 %
What does this mean?

# Compound interest formula:



$A = P(1 + i)^n$.

$A$ is *amount* after $n$ *periods* when a principal of $P$ dollars invested at an interest rate of $i$ per period.

# Explanation of Bank situation:



interest rate 3.92 %
annual percentage interest: 4%
With calculator.

# Discussion of First Essay



Pull up assignment and discuss.

# Some variants:



\$10,000 at 3.92% compounded daily for 5 years (365 day year)
12165.14103
\$10,000 at 3.92% compounded daily for 5 years (360 day year)
12165.13925
\$10,000 at 4% compounded annually for 5 years
12166.53
END OF Jan 14

# Derivation of Compound Interest formula



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1 period: $A = P(1 + i)$
2 periods: $A = [P(1 + i)](1 + i)$
3 periods: $A = [P(1 + i)(1 + i)](1 + i)$
$n$ periods: $A = P(1 + i)^n$

# Compounding at shorter periods



5 % compounded quarterly for 3 years: $10,000(1 + \frac{.04}{4})^{1}2$
Further calculator examples.

# Continuous Compounding



Let $i$ be the annual interest rate and $n$ be the number of compounding periods per year. The amount after $t$ years is:

$$A = P(1 + \frac{i}{n})^{nt} = P((1 + \frac{i}{n})^n)^t$$

We want to know what happens as $n$ tends to infinity.
What $\lim_{n\to\infty}(1 + \frac{i}{n})^n)$?
Answer: $e^i$.

$$A = Pe^{it}$$

# Rule of 72



**Rule:** The product of interest rate $i$ in percent times the number $t$ of periods for the principal to double is 72:

$$it = 72.$$

Justification: We use continuous interest. We want to know when the amount $A$ is twice the original principal. Thus we want to determine the relation between $i$ and $t$ when

$$2P = Pe^{it}.$$

I.e.

$$2 = e^{it}.$$

Taking the ln of each side of the equation we have: $.693 \approx it$. Taking the interest rate in per cent amounts to multiplying both sides of the equation by 100. And we round off 69.3 to 72 so there will be many divisors.

## Calculating the Limit



This is too technical for the papers you are writing.
**Definition** $\ln x = \int_1^x \frac{1}{t} dt$.
By the fundamental theorem of calculus
**Fact** $\ln' x = \frac{1}{x}$
So $\ln'(1) = 1$.

But by the definition of derivative:

$$\begin{aligned}
1 &= \ln'(1) \\
&= \lim_{n\to\infty} \frac{\ln(1+\frac{1}{n}) - \ln 1}{\frac{1}{n}} \\
&= \lim_{n\to\infty} n(\ln(1+\frac{1}{n})) \\
&= \lim_{n\to\infty} \ln((1+\frac{1}{n})^n)
\end{aligned}$$

So $1 = \ln \lim_{n\to\infty}((1+\frac{1}{n})^n)$
and therefore
$e^1 = \lim_{n\to\infty}((1+\frac{1}{n})^n)$.