Lecture 2: Combinatorial Geometries

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Definition 1 *A* pregeometry *is a set G together with a dependence relation*

$$cl : \mathcal{P}(G) \to \mathcal{P}(G)$$

satisfying the following axioms.

A1. $cl(X) = \bigcup\{cl(X') : X' \subseteq_{fin} X\}$

A2. $X \subseteq cl(X)$

A3. $cl(cl(X)) = cl(X)$

A4. *If $a \in cl(Xb)$ and $a \notin cl(X)$, then $b \in cl(Xa)$.*

If points are closed the structure is called a geometry.

Definition 2 *A geometry is* homogeneous *if for any closed $X \subseteq G$ and $a, b \in G - X$ there is a permutation of G which preserves the closure relation (i.e. an automorphism of the geometry) which fixes X pointwise and takes a to b.*

Exercise 3 *If G is a homogeneous geometry, X, Y are maximally independent subsets of G, there is an automorphism of G taking X to Y.*

Definition 4

1. *The structure M is* strongly minimal *if every first order definable subset of any elementary extension M' of M is finite or cofinite.*
2. *The theory T is strongly minimal if it is the theory of a strongly minimal structure.*
3. *$a \in \mathrm{acl}(X)$ if there is a first order formula with finitely solutions over X which is satisfied by a.*

Definition 5 *Let X, Y be subsets of a structure M. An elementary isomorphism from X to Y is* 1-1 *map from X onto Y such that for every first order formula $\phi(\mathbf{v})$, $M \models \phi(\mathbf{x})$ if and only if $M \models \phi(f\mathbf{x})$.*

Exercise 6 *Find X, Y subsets of a structure M such that X and Y are isomorphic but not elementarily isomorphic.*

Exercise 7 *Let X, Y be subsets of a structure M. If f takes X to Y is an elementary isomorphism, f extends to an elementary isomorphism from $\mathrm{acl}(X)$ to $\mathrm{acl}(Y)$.*

Exercise 8 *Show a complete theory T is strongly minimal if and only if it has infinite models and*

1. *algebraic closure induces a pregeometry on models of T;*
2. *any bijection between acl-bases for models of T extends to an isomorphism of the models.*

Exercise 9 *A strongly minimal theory is categorical in any uncountable cardinality.*