# Eliminating Exchange

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We are in the situation of the qm-excellence implies categoricity theorem:

Let $\boldsymbol{K}$ be a quasiminimal excellent class and suppose $H, H' \in \boldsymbol{K}$ satisfy the countable closure condition. Let $\mathcal{A}, \mathcal{A}'$ be cl-independent subsets of $H, H'$ with $\mathrm{cl}(\mathcal{A}) = H$, $\mathrm{cl}(\mathcal{A}') = H'$, respectively, and $\psi$ a bijection between $\mathcal{A}$ and $\mathcal{A}'$.

Fix a countable subset $W$ and write $\mathcal{A}$ as the disjoint union of $\mathcal{A}_0$ and a set $\mathcal{A}_1$; without loss of generality, we can assume $\psi$ is the identity on $\mathcal{A}_0$ and work over $G = \mathrm{cl}(\mathcal{A}_0)$. We may write $\mathrm{cl}^*(X)$ to abbreviate $\mathrm{cl}(\mathcal{A}_0 X)$.

**Exercise 1** *Do not assume exchange. Suppose* $\mathrm{cl}(a) \cap \mathrm{cl}(b) \neq \emptyset$ *where* $a$ *and* $b$ *are independent. Show that for any* $c$ *independent from* $a$*,* $\mathrm{cl}(a) \cap \mathrm{cl}(b) = \mathrm{cl}(a) \cap \mathrm{cl}(c)$*. Conclude that there is a hull* $H_0$ *of the empty set such that for any* $a, b \in \mathcal{A}$*,* $\mathrm{cl}(a) \cap \mathrm{cl}(b) = H_0$*. Conclude further that if for each* $a \in \mathcal{A}$*, we can choose for each* $a$ *a partial* $G$*-monomorphism* $\psi_a$ *with domain* $\mathrm{cl}^*(a)$*, such that* $\bigcup_{a \in \mathcal{A}} \psi_a$ *is a partial* $G$*-monomorphism.*

Now we want to extend this to arbitrary finite independent sets.

???? I thought I knew what came next but ??? Prove that for a finite $X \subset \mathcal{A}_1$, there is a hull $H_X$ which is the intersection of $\mathrm{cl}(Y)$ for all $Y \subset \mathcal{A}_1$ with $|Y| = |X| + 1$ and