Demonstration vrs Deduction

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Acknowledgement

These notes paraphrase work of John Corcoran but the basic distinctions are due to Aristotle.

Aristotle

Demonstrative logic is the study of demonstration (conclusive proof) as opposed to persuasion or even probable proof. Demonstration produces knowledge.

Deductive Logic

Deduction is the process of extracting information implied by given premises regardless of whether those premises are known to be true or even whether they are true.

Demonstration

A demonstration has three parts:

1. Premises
2. Deduction
3. Conclusion

Deduction (Proof)

Corcoran

Deduction is the process of extracting information implied by given premises regardless of whether those premises are known to be true or even whether they are true.

After all, even false propositions imply logical consequences; we can determine that a premise is false by deducing from it a consequence we already know to be false.

A deduction from unknown premises also produces knowledge of the fact that its conclusion follows logically from (is a consequence of) its premises - not just from knowledge of the truth of its conclusion.

The role of diagrams

Aristotle also discovered that deduction is non-empirical in the sense that external experience is irrelevant to the process of deducing a conclusion from premises. Diagrams, constructions, and other aids to imagining or manipulating subject matter are irrelevant hindrances to purely logical deduction.

In fact, in the course of a deduction, any shift of attention from the given premises to their subject matter risks the fallacy of premise smuggling – information not in the premises but intuitively evident from the subject matter might be tacitly assumed in an intermediate conclusion. This would be a non sequitur, vitiating the logical cogency of reasoning even if not engendering a material error.

What is a proof?

For Aristotle, a demonstration begins with premises that are known to be true and shows by means of chaining of evident steps that its conclusion is a logical consequence of its premises.

Thus, a demonstration is a step-by-step deduction whose premises are known to be true.

What is geometry?

Old View

Until the 19th century it was thought that geometry was the deduction of truths from unassailable premises.

These premises were Euclid's Axioms (common notions) and Postulates (geometric assumptions).

What is geometry?

New View

Geometry is the deduction of conclusion from a certain set of geometric hypotheses.

These hypotheses might be Euclidean, spherical, hyperbolic, geometry etc.
Whether these geometrical hypotheses are "true" is not a mathematical question.

Axiomatic System

An axiomatic system provides a small understandable set of postulates from which all results in the system can be deduced.

Ideally these postulates should be independent.
They isolate the truly basic ideas of the subject.

Hilbert and Euclid (with gaps) provide axiom systems that meet these two criteria. Most modern geometry texts do not.

Local Proof

Local proof or local axiomatics just makes clear the premises of a particular argument without attempting to show the overall structure of the subject.

Definitions

A definition is an abbreviation.

We have some concept X which we can describe in terms of earlier notions and we say X holds iff this collection of conditions hold.
Keypoints:

1. new notion
2. properties of old notion

Complex Example of Definition

Problem

Define a relation $B((a_1, a_2), (b_1, b_2), (c_1, c_2))$ to interpret **c** is between **a** and **b**.

Intuitive Solution

We must define algebraically a relation B on triples $(a_1, a_2), (b_1, b_2), (c_1, c_2)$ that is satisfied if and only if (c_1, c_2) is on the line segment between (a_1, a_2), and (b_1, b_2).

Formal Definition

Case 1: $a_1 \neq b_1$.

Definition: $B((a_1, a_2), (b_1, b_2), (c_1, c_2))$ if and only if

1. one of the following:
   1. $a_1 \leq c_1 \leq b_1$ if $c_1 < b_1$
   2. or $b_1 \leq c_1 \leq a_1$ if $b_1 < c_1$.
2. and

$$\frac{c_2 - b_2}{c_1 - b_1} = \frac{a_2 - b_2}{a_1 - b_1}$$

(More cumbersome answer in solutions but Dana got this cleaner version.)

Case 2: Vertical line is easier.