Math 592

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October 2, 2007

1 Assignments for October 8

A slightly revised copy of the lecture notes for Oct. 1 is also posted.

1. Let $<$ be a binary relation that satisfies the axioms for order. ($<$ is irreflexive, asymmetric ($a < b \rightarrow \neg b < a$) and transitive. Consider the following conditions.

   (a)
   $$(\forall x)(\exists y)x < y$$

   (b)
   $$(\exists y)(\forall x)x < y$$

   Three of the four statements 1 and 2; 1 and not 2, 2 and not 1, neither 1 nor 2 are possible; one of them is impossible. Give examples of the three that are possible and explain why the other is not.

2. Here is a clarification of the argument I started in class. (I was careless about the domain of the equivalence relation.

   Definition:

   A region is just a set of points in the plane.

   A region X is connected if for any two points A, B in X there is a finite sequence of points $A_1, \ldots A_n$ such that $A_1 = A$, $A_n = B$ and the line segment from A_n to A_{n+1} contains only points that are in X.

   Theorem. The line ℓ through two points A, B divides the plane into two disjoint connected regions.

   Proof. Let Y be the points in the plane that are *not* on ℓ. Define a binary relation on Y by $x \sim y$ iff the segment xy does not intersect ℓ.

   Claim 1. $\sim$ is an equivalence relation. (Proved in class)

   Claim 2. $\sim$ has only two classes.

   Assignment: Prove claim 2. (I used Pasch's axiom and divided into cases using something about betweenness. This is not very long.)

3. Read the article by Baggett and Ehrenfeucht on the web-page. We will discuss that article and return to Harel-Souder next week.