Assignment 6 due 0ct 15

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A. Essentially no one correctly understood the assignment on half-planes. Let me try again. In solving the following problem you may use ONLY

1. the axioms stated before the statement of the half-plane theorem in my Oct. 1 lecture notes (on the web)
2. basic logical principle of deduction and properties of equality
3. the definitions and partial proof outlined below.

Definition: A region is just a set of points in the plane. A region X is connected if for any two points A, B in X there is a finite sequence of points A1, . . .An such that A1 = A, An = B and the line segment from An to An+1 contains only points that are in X.

We want to prove:

Theorem. The line ‘ through two points A,B divides the plane into two disjoint connected regions.

Proof. Let Y be the points in the plane that are not on ‘. Define a binary relation on Y by $x \sim y$ iff the segment xy does not intersect ‘.

Claim 1. $\sim$ is an equivalence relation. (Proved in class)

You may assume claim 1 but if you have any doubts reprove it.

Claim 2. $\sim$ has only two classes.

Assignment: Prove claim 2.

(Hint: The proof will begin by saying. Assume for contradiction that x,y,z are distinct points not on AB that are pairwise inequivalent.

I had to apply Paschs axiom in each of two cases using something about betweenness. The key point is to PROVE that a line cannot intersect all three sides of a triangle.

Further explanation. You are trying to justify the word half-plane. Without a proof we are afraid that a line cuts the plane into 3 (or 4 or 7 or infinitely many) half planes.

B. Let P be a binary relation. We know nothing else. Show that none of the following three sentences is logically implied by the other two. That is for each sentence give a structure in which it is false and the other two are true.

to save time I write Pxy instead of $P(x,y)$.

1.
$$(\forall x)(\forall y)(\forall z)(Pxy \to (Pyz \to Pxz))$$

2.
$$(\forall x)(\forall y)(Pxy \to (Pyx \to x = y))$$

3.
$$(\forall x)(\exists y)Pxy \to (\exists y)(\forall x)Pxy$$

C. Read Wu' article in the links section (go to Wu and Usyskin). You are welcome to read Usyskin now but I am just trying to spread out the reading time.