MCS 261, Nov 12, 2002 Name: ____________________
SAMPLE Hour Exam 2

- ○ Each problem is worth 20 points.
- ○ Write your solutions in your exam book.
- ○ Turn in this sheet along with your solutions.
- ○ Show and explain all of your work.
- ○ **An unjustified answer is not correct!**

---

(1) Prove or provide a counterexample for each of the following:
 (a) If m divides ab then m divides a or m divides b.
 (b) If a prime p divides ab then p divides a or m divides b.

(2) Solve the recursion relation
$$a_n = 6a_{n-1} - 9a_{n-2}$$
with initial conditions $a_0 = 1$ and $a_1 = 3$.

(3) Use mathematical induction to prove the following identity for all natural numbers n.
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

(4) If $x^k + 1$ is prime then k is a power of two.

(5) Find a 1-1 correspondence between $\{\langle a, b\rangle : a, b \in \Re\}$ and the complex numbers, i.e., $\{\langle a + b_i\rangle : a, b \in \Re\}$. What does this say about the relation between the cardinality of the set of reals $\Re$ and the cardinality of the set of complex numbers.