MCS 261, May 3, 2002 Name: ______________________________
Final Exam

- ◦ There are nine (9) problems on this exam.
- ◦ Each problem is worth 20 points.
- ◦ Write your solutions in your exam book.
- ◦ Turn in this sheet along with your solutions.
- ◦ Show and explain all of your work.
- ◦ **An unjustified answer is not correct!**
- ◦ If you use the logical equivalences on the attached page, refer to them by number. Refer to the logical arguments by name.

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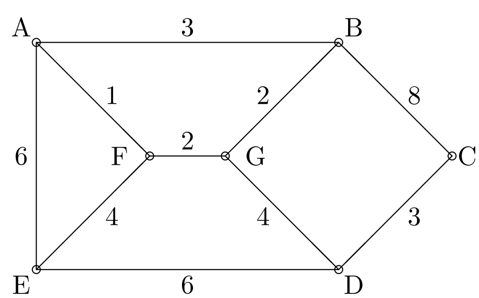
(1) Use the Euclidean algorithm to find $\gcd(-392, 273)$. Show all of your steps.

(2) Let $A = \{1, 2, 3\}$ and $B = \{2, 4\}$.
  (a) Find $A \cup B$, $A \cap B$, $A \backslash B$, $A \oplus B$, and $A \times B$.
  (b) Let $\mathcal{R} = \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 3)\}$. Is $\mathcal{R}$ an equivalence relation on A? Why or why not?

(3) Find a minimum spanning tree of the following graph. You may draw your spanning tree on this sheet. In your exam book, state which algorithm you use and the order in which you add the edges to the tree.



(4) Let p and q be statements. Simplify the following expression completely.

$$[p \wedge (p \rightarrow q)] \rightarrow q$$

(5) (a) Determine how many ways Sally can choose 5 of her 9 closest friends to invite over for dinner, assuming that Alice and Bob are married so if one is invited, they both must be invited. Be sure to explain how you get your answer.

(b) Determine how many ways Sally and 5 of her friends can sit around her circular table, assuming that the group includes Charlie and Donna, who refuse to sit together. Be sure to explain how you get your answer.

(6) Let $f(n) = 3n^2$ and $g(n) = 2n^3$. Use the definition of Big Oh (not a theorem, property, or limit) to prove that $f = \mathcal{O}(g)$ but $g \neq \mathcal{O}(f)$.

(7) Recall that $3\mathbf{Z} + 1 = \{n \in \mathbf{Z} \mid n = 3k + 1 \text{ for some } k \in \mathbf{Z}\}$. Show that $\mathbf{Z}$ and $3\mathbf{Z} + 1$ have the same cardinality.

(8) Ten points are chosen inside a square with side length 3. Prove that two of these points lie within $\sqrt{2}$ of each other.

(9) A sequence is defined by the recursion relation

$$\begin{aligned} a_0 &= 1 \\ a_{n+1} &= 3a_n + 2^{n+1} \text{ for } n \geq 0. \end{aligned}$$

Use mathematical induction to prove that

$$a_n = 3^{n+1} - 2^{n+1}$$

for all $n \geq 0$. *Note:* You do not need strong induction for this proof.