FROM 6.2:

4. (a) The first digit can be anything but 0, so we have 9 choices. The second digit can be anything but the first digit, so 9 choices. The last digit can be anything but the first two, 8 choices. The total is $9 \times 9 \times 8 = 648$.
   (b) There are five choices for the last digit (1, 3, 5, 7, 9). The first digit can be anything but 0 or the last digit so there are 8 choiced. The second digit can be anything but the other two, so 8 choices. The total is $5 \times 8 \times 8 = 320$.

10. We have 8 choices for each of the first two digits, 9 choices for the third digit, and 10 choices for the last four digits. The total is $8^2 \times 9 \times 10^4 = 5,760,000$.

14. (a) There are three choices for the first leg and five for the second leg, so a total of $3 \times 5 = 15$.
   (b) There are 15 routes to get from Cupids to Hearts Desire, as we saw in (a). Next choose one of the 5 routes to go back from Heats Desire to Harbour Grace, then choose one of the 3 routes from Harbour Grace to Cupids. The total is $15 \times 5 \times 3 = 225$.
   (c) Again, there are 15 ways to get to Hearts Desire. Now we can't repeat the road we already used to get back to Harbour Grace, so we have 4 choices. We can't repeat the road we used between Harbour Grace and Cupids so we have 2 choices. The total is $15 \times 4 \times 2 = 120$.

FROM 6.3:

12. Let a_i be the number of words George learns on day i, and let $b_i = a_1 + a_2 + \cdots + a_i$ be the number of words George has learned through the i^{th} day. Since $a_i \geq 1$ for all i, we know that

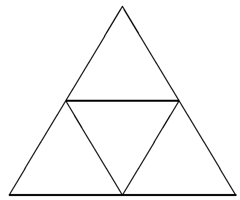
$$1 \leq b_1 < b_2 < \cdots < b_{53} \leq 90$$

   and that

$$16 \leq b_1 + 15 < b_2 + 15 < \cdots < b_{53} + 15 \leq 105.$$

   These are $53 \times 2 = 106$ integers between 1 and 105, so by the Pigeon-Hole Principle, two of them must be the same. We can't have $b_i = b_j$ or $b_i + 15 = b_j + 15$ so there must be some i and some j such that $b_i = b_j + 15$ and $i > j$. Then we have $a_1 + a_2 + \cdots a_i = a_1 + a_2 + \cdots + a_j + 15$. Cancelling like terms gives $a_{j+1} + a_{j+1} + \cdots + a_i = 15$. The consecutive days are those between day $j + 1$ and day i.

17. Divide the triangle into four smaller equilateral triangles with side lengths of $\frac{1}{2}$ as shown below.



By the Pigeon-Hole Principle, of any five points chosen in the triangle, two must lie within the same smaller triangle. The furthest apart these two points can be is the side length $\frac{1}{2}$.

23. Any number between 1 and 200 can be written as $2^k a$ with $k \geq 0$ and a odd such that $1 \leq a \leq 199$. The possible values of a are 100 odd numbers. If we choose 101 integers from $\{1, 2, \ldots, 200\}$, two of those integers, x and y must have the same value of a. That is, two of our chosen numbers are $x = 2^i a$ and $y = 2^j a$ with $i > j$. Thus $x = 2^{i-j}(2^j a) = 2^{i-j} y$ so x is divisible by y.