From 1.2:

12. $n^2 - n + 5 = n(n-1) + 5$. Since n and $n-1$ are consecutive integers, one of them must be even, so $n(n-1)$ is even. Then $n(n-1)+5$ must be odd since an even number plus an odd number is odd.

    Alternate: Using Cases.
    Case 1: n is even. Then $n = 2k$ for some integer k. Then $n^2 - n + 5 = 4k^2 - 2k + 5 = 2(2k^2 - k + 2) + 1$ is odd.
    Case 2: n is odd. Then $n = 2k+1$ for some integer k. Then $n^2 - n + 5 = (4k^2 + 4k + 1) - (2k + 1) + 5 = 4k^2 + 2k + 5 = 2(2k^2 + k + 2) + 1$ is odd.

14. Look at $a^2 - b^2$ in the four suggested cases.
    (i) $a = 2n$ and $b = 2m$ for some integers n, m. Then $a^2 - b^2 = 4n^2 - 4m^2 = 2(2n^2 - 2m^2)$ is even.
    (ii) $a = 2n + 1$ and $b = 2m + 1$ for some integers n, m. Then $a^2 - b^2 = (4n^2 + 4n + 1) - (4m^2 + 4m + 1) = 2(2n^2 + 2n - 2m^2 - 2m)$ is even.
    (iii) $a = 2n$ and $b = 2m + 1$ for some integers n, m. Then $a^2 - b^2 = 4n^2 - (4m^2 + 4m + 1) = 2(2n^2 - 2m^2 - 2m) + 1$ is odd.
    (iv) $a = 2n + 1$ and $b = 2m$ for some integers n, m. Then $a^2 - b^2 = 4n^2 + 4n + 1 - 4m^2 = 2(2n^2 + 2n - 2m^2) + 1$ is odd. From this we see that $a^2 - b^2$ is odd exactly when one of a and b is odd and the other is even. This is the necessary and sufficient condition because $a^2 - b^2$ is odd if and only if one of a and b is odd and the other is even.

19. Proof by contradiction. We begin by assuming the negation of what we want to prove, that is, that there *is* a smallest positive real number, r. Then consider $\frac{r}{2}$. This is a real number (since a real divided by a real is real) and is positive (since a positive number divided by a positive number is positive). In addition, $\frac{r}{2} < r$. But this contradicts our assumption that r was the smallest. Thus the assumption cannot be true and there is no smallest positive real number.

25. (d) Counterexample: If $a = \sqrt{2}$ and $b = -\sqrt{2}$, then $a + b = 0$, which is rational, so the hypothesis is true. However, a and b are not rational (we proved this fact in class). So the statement is false.

From 1.3:

1. (e) The truth table is:

p	q	r	$q \to r$	$p \to (q \to r)$	$p \wedge q$	$(p \wedge q) \vee r$	$(p \to (q \to r)) \to ((p \wedge q) \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	F	F
F	T	T	T	T	F	T	T
F	T	F	F	T	F	F	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	F	F