FROM 1.4:

1. Verification of 6i:

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

The columns for $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are the same so they are logically equivalent.

2. (c)

$$
\begin{array}{ll}
 & [(p \to q) \vee (q \to r)] \wedge (r \to s) \\
(13) & [(\neg p \vee q) \vee (\neg q \vee r)] \wedge (r \to s) \\
(3) & [((\neg p \vee q) \vee \neg q) \vee r] \wedge (r \to s) \\
(3) & [(\neg p \vee (q \vee \neg q)) \vee r] \wedge (r \to s) \\
(9) & [(\neg p \vee 1) \vee r] \wedge (r \to s) \\
(7) & [1 \vee r] \wedge (r \to s) \\
(7) & 1 \wedge (r \to s) \\
(7) & r \to s
\end{array}
$$

4. (c)

$$
\begin{array}{ll}
 & \neg(p \leftrightarrow q) \\
(12) & \neg[(p \to q) \wedge (q \to p)] \\
(13) & \neg[(\neg p \vee q) \wedge (\neg q \vee p)] \\
(6) & \neg(\neg p \vee q) \vee \neg(\neg q \vee p) \\
(6) & (\neg(\neg p) \wedge \neg q) \vee (\neg(\neg q) \wedge \neg p) \\
(5) & (p \wedge \neg q) \vee (q \wedge \neg p) \\
(4) & [(p \wedge \neg q) \vee q] \wedge [(p \wedge \neg q) \vee \neg p] \\
(4) & [(p \vee q) \wedge (\neg q \vee q)] \wedge [(p \vee \neg p) \wedge (\neg q \vee \neg p)] \\
(9) & [(p \vee q) \wedge 1] \wedge [1 \wedge (\neg q \vee \neg p)] \\
(7) & (p \vee q) \wedge (\neg q \vee \neg p) \\
(5) & (p \vee \neg(\neg q)) \wedge (\neg q \vee \neg p) \\
(13) & (\neg q \to p) \wedge (p \to \neg q) \\
(12) & p \leftrightarrow \neg q
\end{array}
$$

5. First, we show the statements are equivalent.

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \to q$	$(p \wedge \neg q) \to \neg p$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	F	T	T

The statements are equivalent since their columns of the truth table are the same. You might notice that these columns are also the same as the truth table for $p \to q$, so these statements are equivalent to that. Here is how to simplify to get it using the rules.

$$\begin{array}{ll} & (p \wedge \neg q) \to q \\ (13) & \neg(p \wedge \neg q) \vee q \\ (6) & (\neg p \vee \neg(\neg q)) \vee q \\ (5) & (\neg p \vee q) \vee q \\ (3) & \neg p \vee (q \vee q) \\ (1) & \neg p \vee q \\ (13) & p \to q \end{array}$$

FROM 1.5:

3. (c) The second and third premises can be rewritten as $p \to r$ and $r \to s$, respecively. Using the chain rule, this gives the conclusion $p \to s$. Rewrite this as $\neg p \vee s$. We have now show that our argument is equivalent to the argument

$$\begin{array}{c} p \vee q \\ \neg p \vee s \\ \hline q \vee s \end{array}$$

This argument is resolution, so we know it is valid.

4. (c) Rewrite the premises as implications using property 13. I have numbered the premises for reference.

$$\begin{array}{cc} (1) & q \to p \\ (2) & (t \vee s) \to (p \vee r) \\ (3) & r \to (t \vee s) \\ (4a) & p \to (t \vee s) \\ (4b) & (t \vee s) \to p \\ \hline & (q \vee r) \to (p \vee r) \end{array}$$

Combining (1) and (4a) with the chain rule gives (5) $q \to (t \vee s)$. Combine this with (3) to get (6) $(q \vee r) \to (t \vee s)$ ((5) means that if q is true, then so is $(t \vee s)$, (3) means that if r is true, then so is $(t \vee s)$; thus if one of q or r is true, then so is $(t \vee s)$.) Now combine (6) and (2) with the chain rule to get the conclusion. This argument is valid.

(d) This argument is not valid. When p is false and q, r, and s are true, all of the premises are true but the conclusion is false. (Note: these are not the only truth values for which this is true.) I found these by trying to make the conclusion false. I wanted $(p \wedge q)$ to be false and $[(q \wedge r) \vee s]$ to be true. This was one way that worked and also all the premises came out to be true.

5. (d) Let p be the statement "I stay up late at night," and q be the statement "I am tired in the morning." Then, in symbols, the argument is

$$\begin{array}{c} p \to q \\ \neg p \\ \hline \neg q \end{array}$$

The following row of the truth table shows that this argument is not valid since in this case both premises are true, but the conclusion is false.

p	q	$p \to q$	$\neg p$	$\neg q$
F	T	T	T	F

8. (a) In determining the validity of arguments we are concerned only with the cases in which all of the premises are true. For the premise $p \wedge q$ to be true, the values of p and q both have to be true, and vice versa so we can use p and q as premises since they are both true if and only if $p \wedge q$ is true.

(b) Replace $p \wedge q$ with the premises p and q.

$$\begin{array}{cc} (1) & p \\ (2) & q \\ (3) & p \to r \\ (4) & s \to \neg q \\ \hline & \neg s \wedge r \end{array}$$

Combine (1) and (3) with modus ponens to get (5) r. Combine (2) and (4) with modus tollens to get (6) $\neg s$. The argument is now equivalent to

$$\begin{array}{cc} (5) & r \\ (6) & \neg s \\ \hline & \neg s \wedge r \end{array}$$

This is obviously a valid argument.