From 1.4:

7. (a) True. Every set is a subset of itself.
   (b) True. The empty set is a subset of every set.
   (c) False. The empty set has no elements, so $\emptyset$ is not a member of it.
   (d) True. The set $\{\emptyset\}$ contains one element and $\emptyset$ is it.

10. (b) Suppose $A = \{a\}$. Then the power set of A is $\mathcal{P}(A) = \{\emptyset, \{a\}\}$. The power set of $\mathcal{P}(A)$ is $\mathcal{P}(\mathcal{P}(A)) = \{\emptyset, \{\emptyset\}, \{\{a\}\}, \{\emptyset, \{a\}\}\}$, which has 4 elements. If you get confused seeing this, think of $\emptyset$ as x and $\{a\}$ as y and find the power set of $\{x, y\}$ then plug back in.

12. (c) The statement is true. We use the direct method of proof. Assume that $A \subsetneqq B$ and $B \subsetneqq A$, and prove that $A \subsetneqq C$. We first show that $A \subseteq C$. Let $x \in A$. Since $A \subseteq B$, then $x \in B$. Since $B \subseteq C$, then $x \in C$. So if $x \in A$, then $x \in C$, so $A \subseteq C$. We now need to show that $A \neq C$. $A \subsetneqq B$, so there is an element, call it y, such that $y \in B$ but $y \notin A$. Since $B \subseteq C$, $y \in C$ but $y \notin A$, so $A \neq C$.

    (e) This statement is false in general. The following is a counterexample. Let $A = \{1, 2\}$, $B = \{\{1, 2\}, 3, 4\}$, and $C = \{\{1, 2\}, 3, 4, \emptyset\}$. Then A is is a member of B and B is a subset of C, but A is not a subset of C. A is an *element* of C, but not a *subset*.

From 1.4:

1. (b) $A \cup C = \{-2, 0, 1, 2, 3, 4, 5, 6\}$
   $B \cap C = \{0, 2\}$
   $B \backslash C = \{-1, 1, 3, 4, 5\}$
   $A \oplus B = \{-1, 6\}$
   $C \times (B \cap C) = \{(-2, 0), (-2, 2), (0, 0), (0, 2), (2, 0), (0, 2)\}$
   $(A \backslash B) \backslash C = \{6\} \backslash C = \{6\}$
   $A \backslash (B \backslash C) = \{2, 6\}$
   $(B \cup \emptyset) \cap \{\emptyset\} = B \cap \{\emptyset\} = \emptyset$

9. (c) $M \cap P^c \neq \emptyset$
   (d) $(CS \backslash T) \subseteq P$
   (e) $(M \cup CS) \cap P \subseteq T^c$

18. (a) One possible counterexample: $A = \{0, 1, 2\}$, $B = \{1, 2, 3\}$, $C = \{2, 3, 4\}$. Then
    $$A \cup (B \cap C) = \{0, 1, 2\} \cup \{2, 3\} = \{0, 1, 2, 3\}$$
    but
    $$(A \cup B) \cap C = \{0, 1, 2, 3\} \cap \{2, 3, 4\} = \{2, 3\}.$$
    (b) We show that each set is a subset of the other.
    First we show that $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. Let $x \in A \cup (B \cap C)$. Then $x \in A$ or $x \in B \cap C$. Use cases.

Case 1. $x \in A$. Then $x \in A \cup B$ and $x \in A \cup C$ so $(x \in A \cup B) \cap (A \cup C)$.
Case 2. $x \in B \cap C$. Then $x \in B$ and $x \in C$ so $x \in A \cup B$ and $x \in A \cup C$, thus $x \in (A \cup B) \cap (A \cup C)$.
So whenever $x \in A \cup (B \cap C)$, $x \in (A \cup B) \cap (A \cup C)$, so $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

Next we show that $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. Let $x \in (A \cup B) \cap (A \cup C)$. Then $x \in (A \cup B)$ and $x \in (A \cup C)$. Let $\mathcal{A}$ be the statement "$x \in A$", $\mathcal{B}$ be the statement "$x \in B$", and $\mathcal{C}$ be the statement "$x \in C$". Then we have $(\mathcal{A} \vee \mathcal{B}) \wedge (\mathcal{A} \vee \mathcal{C})$. By the distributive logical equivalence, this is equivalent to $\mathcal{A} \vee (\mathcal{B} \wedge \mathcal{C})$, which translates back to $x \in A$ or $x \in B$ and $x \in C$, that is, $x \in A$ or $x \in B \cap C$, so $x \in A \cup (B \cap C)$. So whenever $x \in (A \cup B) \cap (A \cup C)$, $x \in A \cup (B \cap C)$, so $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$. This completes the proof.

19. We assume that $(A \cup B)^c = A^c \cap B^c$ and prove $(A \cap B)^c = A^c \cup B^c$. We will use the fact that $(x^c)^c = X$. Let $X = A^c$ and $Y = B^c$ so $A = X^C$ and $B = Y^C$. Then we get the following sequence of equalities, applying DeMorgan's first law to $X^c \cap Y^c$.

$$(A \cap B)^c = [X^c \cap Y^c]^c = [(X \cup Y)^c]^c = X \cup Y = A^C \cup B^c.$$