FROM 2.3:

3. (d) The relation is reflexive since for each person a, a has the same parents as him/herself. The relation is symmetric since if a has the same parents as b, then b has the same parents as a. The reflection is transitive since if a has the same parents as b and b has the same parents as c, then a has the same parents as c.

5. (b) One possibility is $\mathcal{R} = \{(1,1),(1,2),(2,2),(2,3),(3,3)\}$. This is reflexive since for all $a \in A$, the pair (a,a) is in the relation. It is not symmetric because $(1,2)$ is in the relation, but $(2,1)$ is not. It is not transitive since $(1,2)$ and $(2,3)$ are in the relation, but $(1,3)$ is not.

   (c) One possibility is $\mathcal{R} = \{(1,2),(2,1)\}$. This is symmetric because whenever (a,b) is in the relation, (b,a) is also in the relation. Is is not reflexive since none of the pairs (a,a) appear. It is not transitive because even though $(1,2)$ and $(2,1)$ are in the relation, $(1,1)$ is not (note that here $a = 1$, $b = 2$, and $c = 1$).

   (d) One possibility is $\mathcal{R} = \{(1,2),(2,3),(1,3)\}$. This is transitive because whenever (a,b) and (b,c) are in the relation, (a,c) is also in the relation. It is not reflexive since none of the pairs (a,a) appear. It is not symmetric because even though $(1,2)$ is in the relation, $(2,1)$ is not.

6. It is possible for a relation to be both symmetric and antisymmetric. Here is the reasoning about what such relations must look like. Let $\mathcal{R}$ be such a relation. Suppose there is a pair (a,b) in $\mathcal{R}$. Since $\mathcal{R}$ is symmetric, the pair (b,a) must also be in the relation. But since $\mathcal{R}$ is also antisymmetric and both (a,b) and (b,a) are in the relation, we must have that $a = b$. So the only pairs in $\mathcal{R}$ are of the form (a,a). So all relations on a set A that are both symmetric and antisymmetric are subsetsof $\{(a,a)|a \in A\}$.

9. (e) This relation is reflexive because for all real numbers a, $a - a = 0 \leq 3$, so (a,a) is in the relation. It is not symmetric because $(1,5)$ is in the relation $(1 - 5 = -4 \leq 3)$ but $(5,1)$ is not in the relation $(5 - 1 = 4 \not\leq 3)$. It is not antisymmetric because $(1,3)$ and $(3,1)$ are both in the relation, but $1 \neq 3$. It is not transitive because $(1,3)$ and $(3,5)$ are both in the relation, but $(1,5)$ is not.

FROM 2.4:

5. First, we show that the relation is has all three properties required by equivalence relations. We will use the fact that we proved earlier in the semester that a^2 is even (or odd) if and only if a is even (or odd), so $a^2 + b$ is even if and only if a and b are both even or both odd. The relation is symmetric because for all $a \in \mathbf{N}$, $a^2 + a$ is even. The relation is symmetric because if $a^2 + b$ is even, then a and b are both even or both odd, so $b^2 + a$ is even. The relation is transitive because if $a^2 + b$ and $b^2 + c$ are both

even, that means a and b are either both even or both odd and b and c are either both even or both odd, so a, b, and c are either all even or all odd, so $a^2 + c$ is even. Thus the relation is an equivalence relation since it is reflexive, symmetric, and transitive.

Now we want to find the quotient set of the relation, so we need to know what the equivalence classes are. First, let's figure out what the equivalence class of 0 is. $\overline{0} = \{x \in \mathbf{N} | 0^2 + x \text{ is even}\} = \{x \in \mathbf{N} | x \text{ is even}\}$. So all even numbers are in the same equivalence class. The equivalence class of 1 is $\overline{1} = \{x \in \mathbf{N} | 1^2 + x \text{ is even}\} = \{x \in \mathbf{N} | x \text{ is odd}\}$. So there are two equivalence classes, the even natural numbers ($2\mathbf{Z}$) and the odd natural numbers ($2\mathbf{Z} + 1$), so the quotient set is $\{2\mathbf{Z}, 2\mathbf{Z} + 1\}$.

12. (b) The given relation is an equivalence relation. It is reflexive because for all circles a, a has the same center as a so $a \sim a$. It is symmetric because if $a \sim b$, that means a has the same center as b, so b has the same center as a so $b \sim a$. It is transitive because if $a \sim b$ and $b \sim c$, that means a has the same center as b and b has the same center as c, so a has the same center as c so $a \sim c$.

13. (b) $\mathcal{R} = \{(1,1), (2,2), (3,3), (3,4), (4,3), (4,4), (5,5)\}$.

From 3.1:

1. (d) This is not a function because there is more that one b such that $(1, b)$ is in the function. In addition, there are no b's such that $(2, b)$, $(3, b)$, and $(4, b)$ are in the function.

18. (b) I am going to assume that the target of this function is the real numbers. We can't have the denominator be 0 so $x = 1$ is not in the domain. We also can't have the square root of a negative number so any x with $1 - x < 0$, that is, $x > 1$, can not be in the domain. All other real numbers can be in the domain. Thus the domain is all $x < 1$, that is, the interval $(-\infty, 1)$. This function is never negative or 0, so these numbers are not in the range. For any $y > 0$, there is an $x = 1 - \frac{1}{y^2} < 1$ in the domain such that $f(x) = y$ so the range is all $y > 0$, or the interval $(0, \infty)$.