From 4.1:

5. (a) $5286 = 278 \cdot 19 + 4$ so $q = 278$ and $r = 4$.
(b) $-5286 = -279 \cdot 19 + 15$ so $q = -279$ and $r = 15$.
(c) $5286 = -278(-19) + 4$ so $q = -278$ and $r = 4$.
(d) $-5286 = 279(-19) + 15$ so $q = 279$ and $r = 15$.

8. (a) The domain of f is $\mathbf{Z}$, the integers. The range of f is all of the possible remainders, $\{0, 1, 2, \ldots, n-1\}$.
(b) f is not one-to-one because $f(0) = 0$ since $0 = 0 \cdot n + 0$ and $f(n) = 0$ since $n = 1 \cdot n + 0$. Thus $f(0) = f(n)$ but $0 \neq n$.
(c) f is not onto because the range ($\{1, 2, \ldots, n-1\}$) is not equal to the target $\mathbf{N} \cup \{0\}$.

10. (b) Begin in the following way.

$$\begin{aligned}
57483 &= 28741 \cdot 2 + 1 \\
28741 &= 14370 \cdot 2 + 1 \\
14370 &= 7185 \cdot 2 + 0 \\
7185 &= 3592 \cdot 2 + 1 \\
3592 &= 1796 \cdot 2 + 0 \\
1796 &= 898 \cdot 2 + 0 \\
898 &= 449 \cdot 2 + 0 \\
449 &= 224 \cdot 2 + 1 \\
224 &= 112 \cdot 2 + 0 \\
112 &= 56 \cdot 2 + 0 \\
56 &= 28 \cdot 2 + 0 \\
28 &= 14 \cdot 2 + 0 \\
14 &= 7 \cdot 2 + 0 \\
7 &= 3 \cdot 2 + 1 \\
3 &= 1 \cdot 2 + 1 \\
1 &= 0 \cdot 2 + 1
\end{aligned}$$

Then the binary representation is the remainders written in the reverse order:

$$(57483)_{10} = (1110000010001011)_2.$$

We can find the octal representation in a similar way or we can use the binary representation. Notice that the binary representation tells us that

$$57483 = 2^{15} + 2^{14} + 2^{13} + 2^7 + 2^3 + 2^1 + 2^0.$$

Remember that $8 = 2^3$, so rewrite in terms of 2^3.

$$\begin{aligned}
57483 &= (2^3)^5 + 2^2 \cdot (2^3)^4 + 2 \cdot (2^3)^4 + 2 \cdot (2^3)^2 + (2^3) + (2+1) \\
&= 8^5 + (4+2) \cdot 8^4 + 2 \cdot 8^2 + 8^1 + 3 \cdot 8^0 \\
&= (160213)_8
\end{aligned}$$

To get the hexadecimal representation, use the fact that $16 = 2^4$. Recall that in hexadecimal, 10 is represented by A, 11 by B, 12 by C, 13 by D, 14 by F, and 15 by G.

$$\begin{aligned} 57483 &= 2^{15} + 2^{14} + 2^{13} + 2^7 + 2^3 + 2^1 + 2^0 \\ &= 2^3 \cdot (2^4)^3 + 2^2 \cdot (2^4)^3 + 2 \cdot (2^4)^3 + 2^3 \cdot (2^4)^1 + (2^3 + 2^1 + 2^0) \\ &= (8 + 4 + 2)16^3 + 8 \cdot 16^1 + (8 + 2 + 1)16^0 \\ &= (F08B)_{16} \end{aligned}$$

(c) $185,178 = (101101001101011010)_2 = (551532)_8 = (2B35A)_{16}$.

FROM 4.1:

9. (b) This statement is false. Let $a = 6$, $b = 24$, and $c = 8$. Then $a|b$ and $c|b$ but $ac = 48 \not| b$.
(c) This statement is true. If $a|b$, then $b = ak$ for some integer k. So $bc = (ak)c = a(kc)$ and kc is an integer so $a|bc$.

11. (j) In this case, b is larger than a, so we start with b. The Euclidean algorithm gives:

$$\begin{aligned} 54,321 &= 4 \cdot 12,345 + 4941 \\ 12,345 &= 2 \cdot 4941 + 2463 \\ 4941 &= 2 \cdot 2463 + 15 \\ 2463 &= 164 \cdot 15 + 3 \\ 15 &= 5 \cdot 3 + 0 \end{aligned}$$

The last nonzero remainder is 3 so $gcd(12345, 54321) = 3$. To write 3 as a linear combination of a and b, reverse the algorithm.

$$\begin{aligned} 3 &= 2463 - 164 \cdot 15 \\ &= 2463 - 164(4941 - 2 \cdot 2463) \\ &= 329 \cdot 2643 - 164 \cdot 4941 \\ &= 329(12345 - 2 \cdot 4941) - 164 - 4941 \\ &= 329 \cdot 12345 - 822 \cdot 4941 \\ &= 329 \cdot 12345 - 822(54321 - 4 \cdot 12345) \\ &= 3617 \cdot 12345 - 822 \cdot 54321 \end{aligned}$$