Homework 3, selected solutions, Math 261, Spring '02

1.4

2.c). Simplify $[(p \to q) \vee (q \to r)] \wedge (r \to s)$.

$(((\neg p) \vee q) \vee ((\neg q) \vee r)) \wedge (r \to s)$

$((\neg p) \vee q \vee (\neg q) \vee r) \wedge (r \to s)$

$((\neg p) \vee \mathbf{1} \vee r) \wedge (r \to s)$

$\mathbf{1} \wedge (r \to s)$

$r \to s$

5. $[(p \wedge (\neg q)) \to q] \iff [(\neg(p \wedge (\neg q))) \vee q] \iff [((\neg p) \vee q) \vee q] \iff [(\neg p) \vee q]$

$[(p \wedge (\neg q)) \to \neg p] \iff [(\neg(p \wedge (\neg q))) \vee \neg p] \iff [((\neg p) \vee q) \vee \neg p] \iff [(\neg p) \vee q]$

So these are both logically equivalent to $(\neg p) \vee q$.

1.5

1.d)

$$\frac{\begin{array}{c} p \to q \\ (q \vee (\neg r)) \to (p \wedge s) \end{array}}{s \to (r \vee q)}$$

Assume that the argument is not valid. This means that we can find truth values for p, q, r, and s such that the premises are true but the conclusion is false. Since $s \to (r \vee q)$ is false, we must have s true and $r \vee q$ false. But this means both r and q are false. Since $p \to q$ is true and q is false, p must be false. But then $q \vee (\neg r)$ is true and $p \wedge s$ is false, contradicting the truth of $(q \vee (\neg r)) \to (p \wedge s)$. Hence we have a contradiction, so the argument is valid.

5. c) Let p= "I stay up late at night.", and q = "I am tired in the morning." Then the given argument is $\dfrac{\begin{array}{c} p \to q \\ \neg q \end{array}}{\neg p}$

which is valid by modus tollens.

d) The given argument is $\dfrac{\begin{array}{c} p \to q \\ \neg p \end{array}}{\neg q}$

This is not valid, since when $p = F$ and $q = T$, the hypotheses are true, but the conclusion $\neg q = F$ is false.