Homework 4, selected solutions, Math 261, Spring '02

2.1
3.

a. $\{1,2\}, \{1,2,3\}, \{1,2,4\}, \{1,2,3,4\}$

b. $\emptyset, \{1\}, \{2\}, \{1,2\}$

c. $\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}$

d. $\{3\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}$

e. $\{1,2,3\}, \{1,2,4\}, \{1,2,3,4\}$

f. $\emptyset, \{1\}, \{2\}$.

14.

a. In book.

b. True. ($\rightarrow$) Suppose $A \subseteq B$. We prove $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. For this, let $X \in \mathcal{P}(A)$. Therefore, X is a subset of A; that is, every element of X is an element of B. Since $A \subseteq B$, every element of X must be an element of B. So $X \subseteq B$; hence, $X \in \mathcal{P}(B)$.

($\leftarrow$) Conversely, assume $\mathcal{P}(A) \subseteq \mathcal{P}(B)$. We must prove $A \subseteq B$. For any set A, we know that $A \subseteq A$ and, hence, $A \in \mathcal{P}(A)$. Here, with $\mathcal{P}(A) \subseteq \mathcal{P}(B)$, we have, therefore, $A \in \mathcal{P}(B)$; that is, $A \subseteq B$, as desired.

c. The double implication here is false because the implication $\rightarrow$ is false. If $A = \emptyset$, then $\mathcal{P}(A) = \{\emptyset\}$ and $\{\emptyset\} \neq \emptyset$.

2.2
11.

a. $P \cap E \neq \emptyset$

b. $Z \setminus N \ni 0$

c. $P \subseteq N \cap Z$

d. $(P \setminus \{2\}) \subseteq E^c$