MCS 261, Nov 12, 2002 Name: ______________________________

SAMPLE Hour Exam 2 : Some solutions

1. Solve the recursion relation

$$a_n = 6a_{n-1} - 9a_{n-2}$$

with initial conditions $a_0 = 1$ and $a_1 = 3$.

Solution:

Applying the algorithm: solve $x^2 - 6 + 9 = 0$ which has double roots at $x = 3$. So the solution has the form

$$a_n = c_1(3^n) + c_2n(3^n).$$

So we try to solve the system of equations:

$$1 = a_0 = c_1(3^0) + c_20(3^0)$$
$$3 = a_1 = c_1(3^1) + c_23(3^1)$$

and we get $c_1 = 1$ (Don't forget to substitute 0 for the n in $c_2n(3^n)$.) and $3 = 3c_1 + 3c_2$ so $c_2 = 0$.

The solution is $a_n = 3^n$.

4. If $2^k + 1$ is prime then k is a power of two.

We show the contrapositive by showing that if k is not a power of two then $x^k + 1$ factors:

If k is odd the factorizations is $x^k + 1 = (x + 1)(x^{k-1} - x^{k-2} + \ldots 1)$.

If k is twice an odd number, the factorizations is

$$x^k + 1 = (x^2 + 1)(x^{k-2} - x^{k-4} + \ldots 1).$$

Thus, if k is not a power of 2 (i.e if k has an odd factor), either $2 + 1 = 3$ or $2^2 + 1 = 5$ is a proper factor of $2^k + 1$ and $2^k + 1$ is not prime.