Integers and polynomials
Homework due Sept. 14.

1. List five mental arithmetic problems that illustrate the connections between multiplication of binomials and multiplication of integers.

2. Look up the properties of the real numbers in an algebra text and compare them with the axioms for rings as given in class or on the web site.

3. Describe the purpose of 'factoring' in at most 25 words.

4. The next two problems are just good 12th grade problems if you understand the connection between factoring polynomials and factoring integers.

   (a) Notice that if j is even, $2^j - 1$ is composite unless $j = 2$. We want to find many generalizations of this fact. (Recall that a number is composite if it has a factor other than itself and 1.) Consider numbers of the form $b = 2^j - 1$. A prime of this form is called a Mersenne prime.

      i. Give a property of j (even, odd, prime, composite, etc) that guarantees b is composite.
      ii. Can you find such a property which guarantees that b is prime? (Don't kill yourself on this one.)
      iii. Are there infinitely many primes of the form $c = 2^j + 1$. Yes, no, I don't know.

   (b) Consider numbers of the form $c = 2^j + 1$.

      i. Give a property of j (even, odd, prime, composite, etc) that guarantees c that c is composite.
      ii. Can you find such a property which guarantees that b is prime? (Don't kill yourself on this one.)
      iii. Are there infinitely many primes of the form $c = 2^j + 1$. Yes, no, I don't know.