Modeling II

The asssignment for Nov. 29 will be to hand in careful written solutions to the following problems. Many of them will have been discussed in class tonight. In addition, write a half page reflection explaining the key issues addressed in the assignment for Nov. 15 and how your view of them has changed.

I. What is the difference between a variable and a parameter? What distinguishes the following two equations?

$$y = mx + b$$

$$d = rt + d_0$$

II. Consider the two equations.

$$d = rt + d_0$$

$$d = \frac{at^2}{2} + r_0 t + d_0$$

What do the parameters mean in each equation? Give an explanation for 9th graders for why the equations correctly describe motion. Give an explanation for calculus students for the relation between the two equations.

III. We are now going to reconsider some of the Nov. 15 th assignment. I have added some comments that were suggested by comments of various students.

A. Consider the following problems involving modeling data by a polynomial equation. In each case, give a name for each parameter, give the units of the parameters, and write a sentence or two to give a plausibility argument for the model. (I copied these directly from books except when the problem was so ill-posed I couldn't bear to copy it.)

Read the directions (above). Then, do the following problems.

1. The number of calories C that a person burns performing an activity varies directly with the number of minutes t that the person does the activity. A 160 pound person burns 73 calories by dancing for 20 minutes. Write a linear model that gives C as a function of t.

   Does this really work for every 160 pound person? Units?

2. When the length ℓ of a rectangle is fixed, the area (in square inches) of a rectangle varies directly with its width w in inches. When the width of a rectangle with length ℓ is 12 inches, its area is 36 square inches. Write an equation that gives A as a function of w. (This particular kind of problem is particularly hard to state correctly and precisely. I cleaned up a version that didn't make any literal sense. I am not happy with the result. Write this problem correctly.)

   Questions: What are units? What is the formula supposed to refer to? Analyze carefully the sentence, 'When the width of a rectangle with length ℓ is 12 inches, its area is 36 square inches.'. Several people misunderstood this. Is it ambiguous?

3. Here is data concerning the price of 14 karat gold chains.

length x in inches	16	18	20	24	30
price c in dollars	308	344	380	452	560

   If it makes sense, write an equation giving c as a function of x.

   Remarks: misspelling: carat is 1/5 of a gram; karat is the ratio of pure gold 14 karat gold means 14/24 pure gold;

   caret is the symbol $\hat{}$

   Discuss the parameters in this problem.

4. On any planet the height h (in feet) of a body t seconds after is dropped can be modeled by $h = \frac{-gt^2}{2} + h_0$. A rock dropped from a height of 200 feet takes 3.54 seconds to reach the ground on earth but just under $\sqrt{5}$ seconds on Jupiter. Explain this difference. (Hint: What, exactly are g and h_0?)

   What are the units of g? What is g?

III. Look at a few algebra texts or 'teaching tips' articles and find the silliest example you can of a book giving a mathematical model for what is in fact random data. (If you already did this, just say so on your paper.)

IV. Consider the following data:

gidgets x	-2	-1	0	1	2
gadgets y	-4	6	10	-32	20

Find a fourth degree equation that goes exactly through these five points. Why does it work? Will it always work? This is answered rigorously by the discussion of the Vandermonde determinant below.

Why do we need $k+1$ points to determine a polynomial of degree k? I can make up a degree k polynomial with a given set of k roots.

To show the system of equations that represent the data has a solution, it is enough to show that the matrix below has non-zero determinant if $x_1 = -2, x_2 = -1, x_3 = 0, x_4 = 1, x_5 = 2$. Explain why this is true.

This is the Vandermonde determinant. It is non-zero for any choice of the x_i. If you look at the internet you will see many grungy proofs of this. It would be useful to think about what this means but not terribly important to work out a proof unless you can find one that tells you something.

$$\begin{pmatrix} 1 & x_1 & x_1^2 & x_1^3 & x_1^4 \\ 1 & x_2 & x_2^2 & x_2^3 & x_2^4 \\ 1 & x_3 & x_3^2 & x_3^3 & x_3^4 \\ 1 & x_4 & x_4^2 & x_4^3 & x_4^4 \\ 1 & x_5 & x_5^2 & x_5^3 & x_5^4 \end{pmatrix}$$