1. Last time, I asked you to think of an example of a function $f(x)$ such that $lim_{x\to\infty} f(x)$ does not exist, but $f(x)$ is bounded for all x. Discuss with your group what examples you have thought of, and see what they have in common. Can you think of different examples now?

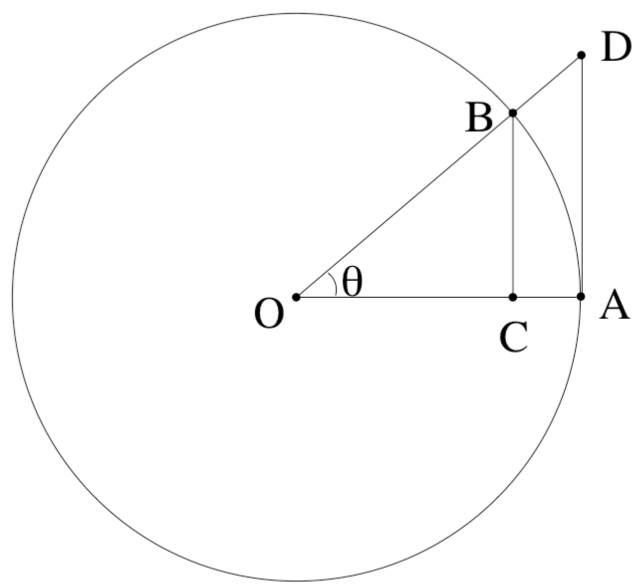
2. Recall the Squeeze Theorem: Suppose the functions f, g, and h satisfy $f(x) \leq g(x) \leq h(x)$ for all values of x near a, except possibly at a itself. Suppose also that $lim_{x\to a} f(x) = lim_{x\to a} h(x) = L$. Then $lim_{x\to a} g(x) = L$.

   (a) Sketch a picture with graphs of some f, g, and h to illustrate this theorem.

   (b) Consider $lim_{x\to 0}(x^3 cos(\frac{3}{x}))$. Why can't you evaluate this limit just by plugging in $x = 0$?

   (c) Write a careful proof using the Squeeze Theorem to show that $lim_{x\to 0}(x^3 cos(\frac{3}{x})) = 0$.

3. Consider the figure



   (a) In the unit circle pictured, find the area of triangle OBC, the area of triangle ODA, and the area of the sector of the circle OBA all as functions of θ.

   (b) Use the areas found above and the Squeeze Theorem to find $\lim_{\theta\to 0} \frac{\sin\theta}{\theta}$. (Hint: you may find it easier to find $\lim_{\theta\to 0} \frac{\theta}{\sin\theta}$ first and then take reciprocals in your work.)

   (c) Use your result to compute $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta}$ algebraically.

   Hint: multiply by a clever form of one.

4. Find examples of functions f and g satisfying the following:

   (a) $\lim_{x\to c} g(x) = 0$ but $\dfrac{f(x)}{g(x)}$ does not have a vertical asymptote at $x = c$.

   (b) $\lim_{x\to c} f(x) = 0$ and $\dfrac{f(x)}{g(x)}$ does have a vertical asymptote at $x = c$.

   (c) $\lim_{x\to c} g(x) \neq 0$ and $\dfrac{f(x)}{g(x)}$ does have a vertical asymptote at $x = c$.

   (d) $\lim_{x\to c} f(x)$ exists and is not zero and $\lim_{x\to c} g(x) = 0$. Does the graph of $\dfrac{f(x)}{g(x)}$ have a vertical asymptote at $x = c$?

5. Let $f(x)$ be a function that has x-intercepts at $x = 0$, $x = 2$, and $x = -2$; y-intercept $(0, 0)$; horizontal asymptote $y = -1$; and vertical asymptotes at $x = 3$ and $x = -3$:

   (a) Graph $f(x)$. Is your graph unique? If so, why? If not, how many distinctive shapes can your graph have?

   (b) As $x \to 3^+$ then $f(x) \to$____?

   (c) As $x \to 3^-$ then $f(x) \to$____?

   (d) As $x \to -3^+$ then $f(x) \to$____?

   (e) As $x \to -3^-$ then $f(x) \to$____?