1. Last time we showed that $\lim_{x\to 0} \dfrac{sinx}{x} = 1$. Use this fact to compute $\lim_{x\to 0} \dfrac{1 - cosx}{x}$ algebraically. (Hint: multiply by a clever form of 1).

2. Determine whether the following statements are true or false and give an explanation or counterexample. Assume a and L are finite numbers.

   (a) If $\lim_{x\to a} f(x) = L$, then $f(a) = L$.

   (b) If $\lim_{x\to a^+} f(x) = L$, then $\lim_{x\to a^-} f(x) = L$.

   (c) If $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = L$, then $f(a) = g(a)$.

   (d) The limit $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ does not exist if $g(a) = 0$.

   (e) If $\lim_{x\to 1^+} \sqrt{f(x)} = \sqrt{\lim_{x\to 1^+} f(x)}$, it follows that $\lim_{x\to 1^-} \sqrt{f(x)} = \sqrt{\lim_{x\to 1^-} f(x)}$.

3. Find examples of functions f and g satisfying the following:

   (a) $\lim_{x\to c} g(x) = 0$ but $\dfrac{f(x)}{g(x)}$ does not have a vertical asymptote at $x = c$.

   (b) $\lim_{x\to c} f(x) = 0$ and $\dfrac{f(x)}{g(x)}$ does have a vertical asymptote at $x = c$.

   (c) $\lim_{x\to c} g(x) \neq 0$ and $\dfrac{f(x)}{g(x)}$ does have a vertical asymptote at $x = c$.

   (d) $\lim_{x\to c} f(x)$ exists and is not zero and $\lim_{x\to c} g(x) = 0$. Does the graph of $\dfrac{f(x)}{g(x)}$ have a vertical asymptote at $x = c$?

   (e) $\lim_{x\to 0} (f(x) + g(x))$ exists but neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exists.

4. Intro to limits at infinity:

   (a) What is $lim_{x\to\infty} \dfrac{1}{x}$?

   (b) What is $lim_{x\to\infty} \dfrac{1}{x^n}$ where $n \geq 1$?

   (c) Use the above facts and some algebra to evaluate $lim_{x\to\infty} \dfrac{3x^3 + 3x + 1}{5x^3 - 2x^2 + 17x - \pi}$. (Hint: rearrange that expression algebraically so that it is written in terms of things like $\dfrac{1}{x^n}$).