1. Formal Infinite Limits

   (a) **Without** looking up the definition in your textbook, discuss what it means to say $lim_{x\to a} f(x) = +\infty$ (where a is a finite number). Try to write a very precise definition that describes this. Draw a graph to illustrate your definition. Label relevant quantities on the x and y axes, and state your definition in terms of those. (This question should take at least several minutes of discussion.)

   (b) Now, look up the formal definition in your textbook (pg 113, or I will write it on the board when we get to this point). Discuss with your group how this compares to your definition from the previous part.

   (c) Use the formal definition from the textbook to write a proof that $\lim\limits_{x\to 1} \dfrac{1}{(x-1)^4} = \infty$, using the following steps (if you get stuck you can also look at Example 6 on pg 114 of your textbook, but try to do this yourselves first):

      i. Scratch work: Let N be an arbitrary positive number, and find a δ in terms of N that makes the definition work.

      ii. Write a step by step proof (this may involve re-writing some of your scratch work). This should be written in complete sentences, with a reason for every statement. (It should literally be complete sentences, remembering that most of the time an = counts as a verb, read "is equal to".)

      iii. Once you've written a proof, read it over. Check that each claim and reason makes sense. Verify that you have proven what you wanted to prove.

      iv. Before moving on, ask me to check your proof. I will either check each group's individually or we will put them on the board, depending on timing.

   (d) Write a proof that $\lim\limits_{x\to 3} \dfrac{1}{(x-3)^6} = \infty$ (again, write scratch work first, then a proof, then check it.)

   (e) (Next time, we will do formal finite limits. After class, try to think about the similarities and differences between those two situations.)

2. Let $g(x) = \sqrt{x}$.

   (a) Write the expression for the slope of the secant line that passes through $(4, g(4))$ and $(4+h, g(4+h))$.

   (b) Calculate the limits of the slopes of the secant lines as as h approaches 0 from the left and the right.

   (c) What can you say about the line tangent to the graph of g at $(4, g(4))$?

3. (a) What is $lim_{x\to\infty} \dfrac{1}{x}$? (Hint: picture the graph.)

   (b) What is $lim_{x\to\infty} \dfrac{1}{x^n}$ where $n \geq 1$?

   (c) Use the above facts and some algebra to evaluate $lim_{x\to\infty} \dfrac{2x^3+3x+1}{5x^3-2x^2+17x-\pi}$ (hint: use algebra to rearrange the expression so that it has terms that look like $\frac{1}{x^n}$)