1. Formal Finite Limits
   Last time, we worked out the formal definition of an infinite limit. Here is the formal definition of a finite limit for a continuous function $f(x)$.
   Definition: $\lim\limits_{x \to a} f(x) = L$ if for every small number $\epsilon > 0$ there is a δ (depending on ϵ) so that: if $|x - a| < \delta$, then $|f(x) - L| < \epsilon$.

   (a) Draw a picture to illustrate this definition. Be careful to keep track of which quantities belong on the x-axis and which belong on the y-axis. Rewrite the definition so that is phrased in terms of intervals instead of absolute values.

   (b) In the case where $f(x) = mx + b$ (i.e. your function is a straight diagonal line), what is the relationship between ϵ and δ and the slope m?

   (c) Prove formally that $\lim\limits_{x \to 2} 3x+1 = 6$. (Just like last time, start with a scratch work section where you find what δ should be in terms of ϵ, and then write a careful proof.)

   (d) Prove formally that $\lim\limits_{x \to 2} x^2 = 4$. (Now that $f(x)$ is not a straight line, you are going to need to use the idea that limits are a local property. We don't care what is happening far away from $x = 2$, so you can restrict your proof to an interval around $x = 2$. How can you use that fact to help your algebra work out?)

   (e) Prove formally that $\lim\limits_{x \to 5} \frac{1}{x} = \frac{1}{5}$

   (f) Read the "proof" on the back of this sheet, and explain what is wrong with it.

2. A function is said to be continuous at the point x_0 if
   (A) $f(x_0)$ is defined
   (B) $\lim_{x \to x_0} f(x)$ exists
   (C) $\lim_{x \to x_0} f(x) = f(x_0)$

   (a) Sketch a graph of a discontinuous function for each of the following:
      i. condition (A) holds, but condition (B) does not
      ii. condition (B) holds, but condition (A) does not
      iii. conditions (A) and (B) both hold, but condition (C) does not

   (b) Classify your examples as removable discontinuities, jump discontinuities, or asymptotes.

   (c) Could you have drawn examples which would have been classified differently?