1. Use the Intermediate Value Theorem to prove that the equation $x^2 = 4^x$ has a solution. Hints/steps:
   (a) Write down the statement of the IVT.
   (b) Check if this problem fits the hypotheses of the theorem. Do you need to change anything in the statement of the problem to make it fit?

2. Derivatives of products:
   (a) Let $f(x) = x + 1$, $g(x) = 3x^2$. Use the definition of the derivative to calculate $f'(x)$, $g'(x)$, $[f(x)g(x)]'$, $f'(x)g'(x)$ and $f'(x)g(x) + f(x)g'(x)$. Which are equal?
   (b) Assume f and g are functions that are differentiable at x. Use the definition of the derivative to prove that $[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$. (Hint: Start with the left hand side, write out the definition of the derivative there, and add and subtract $f(x)g(x+h)$ in the numerator and then break your expression up into two fractions. Or, start with the right hand side, write out the definitions of the derivatives, and see what you can simplify.)
   (c) The rule you just proved is called the Product Rule. Use the Product Rule along with the rules for derivatives of exponential and trig functions (in your textbook) to evaluate the derivative of $e^x sin(x)$.

For the rest of this worksheet, you may use any of the derivative rules printed on the back of the sheet (or the chart in the back of your textbook).

3. Evaluate the following derivatives, stating what rules you used:
   (a) $\frac{d}{dx}6x^5 ln(x)$
   (b) $\frac{d}{dx}cos(x)sin(x)$
   (c) $\frac{d}{dx}\frac{x^7+5x^2+4}{tan(x)}$

4. Find the equation of the tangent line to the given function at $x = a$.
   (a) $y = 2e^x - 4x^2 + 3x; \qquad a = 0$
   (b) $f(x) = 4x^2 - 3x + 1; \qquad a = 2$

5. Determine whether the following statements are true or false, giving an explanation or a counterexample.
   (a) The slope of the tangent line to $f(x) = e^x$ is never zero.
   (b) $f(x) = e^x$ is the only function such that $f'(x) = f(x)$ for all x.
   (c) $\frac{d}{dx}e^x = xe^{x-1}$
   (d) The nth derivative $\frac{d^n}{dx^n}x^3 + 2x = 9$ equals 0 for any integer $n \geq 3$.
   (e) $\frac{d}{dx}\left(\frac{x^2-4}{x+2}\right) = \frac{2x}{1} = 2x$