

Spend 50 minutes doing the following problems. As always, show all your work and reasoning carefully.

1. Let $f(x) = x^2 - 3x$.

(a) Use the definition of the derivative to find $f'(x)$.

(b) Find an equation of the tangent line to the graph of $f(x)$ at the point $x = 2$.

2. Find the derivative of each function. Do not simplify your answer.

(a) $\cos(\sin(x^2 + 3))$ (b) $e^{2x} \ln(\sqrt{x-1})$ (c) $\frac{x^2}{\csc(x)}$

3. Prove that the equation $3^x = x^4$ has at least one (real) solution.

4. Evaluate the following limits or state that they do not exist:

(a) $\lim_{x \rightarrow \infty} \frac{7x^5 + x^4 + 3}{3x^5 + 5x^2 + x + e}$

(b) $\lim_{x \rightarrow \pi/2} \frac{\sin(x) - x^2}{\cos(x)}$

(c) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

5. Let $f(x) = \frac{(x-1)(x-2)(x-5)}{(x-1)(x-3)|x-5|}$.

(a) Find and classify all discontinuities of $f(x)$. (Hint: even if you know the answers from experience with rational functions, to get full credit you must show work that justifies them.)

(b) Does $f(x)$ have any horizontal asymptotes? If so, find them, and if not, explain why.

6. Evaluate the following limit by recognizing it as a derivative: $\lim_{x \rightarrow \frac{\pi}{3}} \frac{\sin(x) - \sin(\frac{\pi}{3})}{x - \frac{\pi}{3}}$