1. Consider the equation $x^2 + y^2 = 1$.

   (a) Sketch the graph.

   (b) Can you represent that graph as the graph of a single function? Why or why not?

   (c) Temporarily pretend $y = f(x)$ is a function of x. Rewrite the equation with $f(x)$ in the place of y. Differentiate the equation term by term, remembering to use the chain rule when needed. This is called implicit differentiation.

   (d) Solve that new equation algebraically for $f'(x)$ (your answer can have an $f(x)$ in it).

   (e) Write your formula for f' in terms of x and y. What does it represent on the graph? What would be a better (or alternate) notation to use here instead of f'?

   (f) Solve the equation $x^2 + y^2 = 1$ for y. (How many functions do you get?). Use this to find a (piecewise) formula for the slope of the tangent line to a circle using explicit differentiation. Show that your answer is equivalent to the one obtained above.

2. A differentiable function $y(x)$ satisfies $x^2 \cos y + \sin y = x$ and $y(1) = 0$. What is $y'(1)$?

3. Sketch graphs of the relations
   (i) $y^2 - x^2 = 0$
   (ii) $y^2 - x^2 = 1$

   (a) Find all continuous functions defined by (i) and (ii). (That means solve for y, and you may get more than one answer.)

   (b) Using implicit differentiation, compute a formula for y' for both (i) and (ii) above.

   (c) Do the functions defined by (i) and (ii) above share a common derivative?

4. (a) Show that if a function f with the property $f'(x) = f(x)$ has inverse g, then $g'(x) = \frac{1}{x}$. (Hint: what is $g(f(x))$?)

   (b) Use this to show that the derivative of $\ln x$ is $1/x$.

   (c) If $\ln(xy) = 2x + 2y$, find $\frac{dy}{dx}$.