

1. Let  $y = \sin^{-1}(x)$ . (Notation: recall that  $\sin^{-1}(x)$  is defined to be the inverse function of  $\sin(x)$ . It is pronounced "sine inverse" and it is also known as  $\arcsin(x)$ . It is not the same as  $1/\sin(x)$ .)

- (a) Solve the equation  $y = \sin^{-1}(x)$  for  $x$ .
- (b) Use implicit differentiation and the derivative rules we already know to find an expression for  $\frac{dy}{dx}$ .
- (c) Use trig identities to write your derivative expression in terms of  $x$  only. (Check that this matches the derivative of  $\sin^{-1}(x)$  given in the back of your book.)

2. Recall that for  $P$  dollars invested for  $t$  years with continuously compounded interest at rate  $r$ , the final amount is

$$A = Pe^{rt}$$

- (a) Solve the formula for  $t$ .
- (b) If I have \$1000 to invest, how long will it take to reach \$2000 if interest at 2.5% is continuously compounded? (Use your previous answer.)
- (c) Find  $\frac{dt}{dP}$ .
- (d) For a small increase in my initial investment amount, how does the time to reach \$2000 change for each additional dollar invested? Use your answer from the previous part and give your answer in days.

3. Verify that

$$\frac{d}{dx} f(x) = f(x) \cdot \frac{d}{dx} [\ln f(x)]$$

Now use this fact to help find  $f'(x)$  for each of the following.

$$(a) f(x) = x^x \quad (b) f(x) = x^{\cos x}$$

4. (a) You are standing at the end of Navy Pier. A plane is traveling in a straight line directly overhead towards Michigan, maintaining a constant altitude of  $h$  feet. Assume that the sun is directly above the plane, so the shadow cast by the plane on the lake is always directly below the plane. Write an expression for the angle of elevation of the plane when the shadow is  $w$  feet away from the end of Navy Pier.
- (b) In the situation above, suppose that the plane is flying at an altitude of 35,000 feet at 830 feet per second. How fast is the angle of elevation changing if it passed overhead three seconds ago?