Name: Worksheet 14 - Related Rates

For each of the following related rates problems, you will probably find it helpful to sketch a picture, label all your variables, and make a list of which quantities are changing and which are constant throughout the problem. Any values that are only true at one particular moment should be listed as such, and should usually not be plugged in until after you have differentiated.

1. A spherical snowball melts in such a way that its surface area decreases at a rate of 2 $\frac{cm^2}{min}$. Find the rate at which the radius is decreasing when the radius is 15 cm. (Recall that the surface area of a sphere with radius r is $4\pi r^2$.

2. (a) You are standing at the end of Navy Pier. A plane is traveling in a straight line directly overhead towards Michigan (that is, the plane passes overhead and then travels away from you over the lake), maintaining a constant altitude of h feet. Assume that the sun is directly above the plane, so the shadow cast by the plane on the lake is always directly below the plane. Write an expression for the angle of elevation of the plane when the shadow is w feet away from the end of Navy Pier.

   (b) In the situation above, suppose that the plane is flying at an altitude of 35,000 feet at 830 feet per second. How fast is the angle of elevation changing if it passed overhead three seconds ago?

3. One sweltering 108° day this past August, Josh and Kyle were cleaning the gutters of their elderly neighbor, Mrs. Macinac, in repentence for having earlier hit a baseball through her dining room window. While Kyle was perched atop a 10 foot ladder, he made the mistake of angering Josh by accusing him of sleeping on the job. In retaliation, Josh began to pull the base of the ladder away from the wall at a rate of $\frac{1}{2}$ ft/sec.

   For the following questions, assume that Kyle's balance is very good, and that the ladder was originally flat against the wall.

   (a) How far does Kyle fall during his first four seconds of motion? The next four? The next four? The next four? The last four? (Use a calculator to simplify your answers, and since we're talking about feet, you can round them to the nearest 10th or 100th of a foot.)

   (b) From what you found in part a), what can you say about the rate at which Kyle is falling?

   (c) How fast is Kyle approaching the ground when Josh has pulled the bottom of the ladder 6 feet from the wall.

   (d) The ladder, the wall, and the ground form a triangle. How fast is the area of the triangle changing when Kyle is 8 feet from the ground? Is the triangle getting larger at this time, or smaller?

4. A baseball diamond is a square with side length 90 ft. The batter hits a ball toward third base at 90 ft/sec, and simultaneously starts running towards 1st base at 30 ft/sec. At what rate is the distance between the ball and the batter changing when she is 30 ft down the line?

5. A streetlight hangs 5 meters above the ground. Regina, who is 1.5 meters tall, walks away from the point under the light at a rate of 2 meters per second.

   (a) How fast is her shadow lengthening when she is 7 meters away from the point under the light? (Hint: Use similar triangles.)

   (b) Suppose Regina has the ability to magically shrink herself. At what rate must she do this to keep her shadow a constant length of 3 meters? Write this as a function of only her distance from the point under the light.