1. First Derivative Test: Your book will present this information to you in Chapter 4, but you can figure it out for yourself without looking it up. (If you have already read about it or discussed it in lecture, try to reconstruct the information without looking it up - that will help you remember it on exams). Let $f(x)$ be a differentiable function. The point $(x_0, f(x_0))$ is called a local maximum if $f(x_0)$ is larger than every other $f(x)$ right around it, that is, on some open interval around x_0. (A local minimum is the same idea - $f(x_0)$ then should be lower than any point on the graph right around it.) Think about what you know about $f'(x)$ and the slope or shape of the graph of $f(x)$. Write down a condition or conditions for what has to be true about $f'(x)$ at or near x_0 in order for $f(x_0)$ to be a local maximum or minimum.

2. Find any critical points of the following functions, and use the first derivative test to determine whether they constitute local maxima or minima. Identify the maximum and minimum values of the function on the interval, if they exist.

   (a) $f(x) = -x^2 - x + 2$ $[-4, 4]$

   (b) $G(x) = (x-1)^{\frac{2}{3}}$ $[0, 8]$

   (c) $h(\theta) = \cos\theta \sin\theta$ $[0, \pi]$ (Double-angle trig identities might come in handy.)

   (d) $z(x) = \dfrac{x^2+1}{x-4}$ $[-1, 4]$ (Be careful.)

3. Use the fact that a quadratic polynomial with roots r and s can be written in the form $f(x) = A(x-r)(x-s)$ for some constant A for these two proofs.

   (a) Suppose that quadratic f has roots r and s. Show that $f'(r) + f'(s) = 0$.

   (b) Show that the critical point of a quadratic occurs midway between its roots.

4. Prove that of all rectangles with given perimeter P, the square has the largest area.

5. (a) Find the closest point on the graph of $f(x) = x^2$ to the point (a, b).

   (b) Show that the line connecting $(0, b)$ to the closest point is normal to the graph at that point. (In this context, "normal" means that they meet in a right angle.)