1. Use the fact that a quadratic polynomial with roots r and s can be written in the form $f(x) = A(x-r)(x-s)$ for some constant A for these two proofs.
   (a) Suppose that quadratic f has roots r and s. Show that $f'(r) + f'(s) = 0$.
   (b) Show that the critical point of a quadratic occurs midway between its roots.
2. Prove that of all rectangles with given perimeter P, the square has the largest area.
3. (a) Find the closest point on the graph of $f(x) = x^2$ to the point (a, b).
   (b) Show that the line connecting $(0, b)$ to the closest point is perpendicular to the graph at that point.
4. Prove that $f(x) = x^3 - 3x + c$ never has two roots in $[0, 1]$ no matter what c is. (Hint: Think about monotonicity.)