

1. Consider the following functions:

$$f(x) = 3x^4 - 8x^3 + 6x^2$$

$$h(x) = xe^{-x^2}$$

$$g(x) = \frac{1}{\cos x + 1} \text{ on } [0, 2\pi]$$

For each of them,

- find the critical points;
  - find the potential inflection points;
  - determine intervals on which the function is increasing, decreasing, concave up, and concave down;
  - determine which points are maximas, minimas, or inflection points
  - sketch the graph.
2. Suppose that a function  $f(x)$  is has  $f'(x) < 0$  and  $f''(x) < 0$  for all  $x$ , with  $f(3) = 5$  and  $f'(3) = -2$ .
- Find an integer  $n$  with  $|f(2) - n| < 1$
  - If  $f(r) = 0$ , find an integer  $k$  with  $|r - k| < 2$

(Hints: Think about what the first and second derivative mean on a graph, and sketch a picture showing what you know about the function. If what you are being asked to find looks confusing, remember that we have discussed several different ways to represent intervals and numbers or variables in them. You may find it helpful to rewrite what you are being asked to find here in a different notation, or in words, or think about what it means on a picture.)

3. Sketch the graph of a function satisfying the following conditions.
- $f'(x) > 0$  and  $f''(x) < 0$  for all  $x$ .
  - $f'(x) < 0$  for  $x < 0$  and  $f'(x) > 0$  for  $x > 0$ , and  $f''(x) < 0$  for  $|x| > 2$  and  $f''(x) > 0$  for  $|x| < 2$ .
4. Find the area of the largest rectangle that can be inscribed in a semi-circle of radius 3.