1. Sketch the graph of a function satisfying the following conditions.
   (a) $f'(x) > 0$ and $f''(x) < 0$ for all x.
   (b) $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$, and $f''(x) < 0$ for $|x| > 2$ and $f''(x) > 0$ for $|x| < 2$.

2. Find the area of the largest rectangle that can be inscribed in a semi-circle of radius 3.

3. (a) Which do you think is bigger, e^π or π^e? Make a guess. Do you have a reason for your guess?
   (b) Let $f(x) = \dfrac{ln(x)}{x}$. Find the maximum of $f(x)$.
   (c) Use this maximum to find and prove which is bigger, e^π or π^e. (Hint: simplify and think about log rules). Was your guess correct?

4. An arbelos is a region enclosed by three mutually tangent semi-circles (two smaller semicircles inside one larger circle, all of which have their centers lying on the same line).
   (a) Sketch an arbelos where the large circle has radius 1, and label the two smaller radii by t and p. Label the distance between the center of the largest circle and the point where the two smaller circles meet by x.
   (b) Write a function for the area of the arbelos in terms of x, t, and p.
   (c) Write equations relating x and t and x and p.
   (d) Substitute so that you have an area function $A(x)$ in terms of x only.
   (e) Find what x should be to maximize the area of the arbelos.
   (f) What does this look like on your picture?

5. Let $g(x) = |x|$.
   (a) For what values of x is $g(x)$ differentiable?
   (b) Show that $g'(x) = x/|x|$.
   (c) Use the chain rule to find a formula for $\frac{d}{dx}(|f(x)|)$, and then use it to find the derivative of $h(x) = |x^2 - 4|$ at $x = 1$.
   (d) Can $\frac{d}{dx}(|f(x)|)$ exist at a point where $f(x) = 0$?