1. Definition: If $g^{\prime}(x)=h(x)$, then we say $h(x)$ is an antiderivative of $g(x)$.
(a) In general, do functions have unique antiderivatives? Can you think of more than one antiderivative for $f(x)=x^{2}$ ?
(b) Suppose $g^{\prime}(x)=h(x)$. How can you write the most general antiderivative for $g(x)$ ? (This is usually what people mean when they say the antiderivative of a function.)
(c) From the basic definition and all the derivatives you already know, you now know a whole list of antiderivatives. To test the concept, for the following functions, write down the most general antiderivative:
i. $f(x)=\cos (x)$
ii. $f(x)=\frac{1}{1+x^{2}}$
iii. $f(x)=x^{n}$ (careful - does this depend on what $n$ is?)
2. Another way to write "the most general antiderivative of $f(x)$ " is to write $\int f(x) d x$. (We will discuss this notation more later). Find the following indefinite integrals (a.k.a. antiderivatives). You may have to put in a little bit of thought - the idea is to find what function you could differentiate in order to get each of these functions. Remember to check your answer by differentiating.
(a) $\int 2 d x$
(b) $\int(9-x)^{2} d x$
(c) $\int \sin (9 x+5) d x$
(d) $\int(4 \theta+\cos 8 \theta) d \theta$
(e) $\int t e^{t^{2}} d t$
3. Suppose $F$ is an antiderivative of $f$, and $G$ is an antiderivative of $g$, that is, $F^{\prime}(x)=f(x)$ and $G^{\prime}(x)=g(x)$. For each of the following, show that it must be true or give a counterexample to show it is false.
(a) If $f=g$ then $F=G$.
(b) If $F$ and $G$ differ by a constant, then $f=g$.
(c) If $f$ and $g$ differ by a constant, then $F=G$.
4. Suppose that $F$ is an antiderivative of $f$, that is, $F^{\prime}(x)=f(x)$.
(a) Show that $\frac{1}{2} F(2 x)$ is the antiderivative of $f(2 x)$.
(b) Find the general antiderivative of $f(k x)$ for any constant $k$.
(c) What is the general antiderivative of $f(k x+a)$ for constants $k$ and $a$ ?
5. A car traveling at $84 \mathrm{ft} / \mathrm{sec}$ begins to decelerate at a constant rate of $14 \mathrm{ft} / \mathrm{sec}^{2}$. After how many seconds does the car come to a stop and how far will the car have traveled before stopping?
