

1. Compute the following integrals by drawing a graph and using what you know about area.

(a) $\int_1^4 x - 2 \, dx$

(b) $\int_0^3 |x - 1| \, dx$

(c) $\int_0^1 \sqrt{1 - x^2} \, dx$

2. Show that $\frac{1}{3} \leq \int_4^6 \frac{1}{x} \, dx \leq \frac{1}{2}$ without evaluating the integral. (Hint: Draw a picture and think about areas.)

3. We know how to compute the average of n numbers: add them up and divide by n . But how can we compute the average of a continuous function? This problem will answer that question. Prof. Bob Loblaw knows that the temperature (heat) of an object varies with time as $H(t) = 3t - 10$ for $0 \leq t \leq 4$.

- (a) Explain why $\frac{1}{8} \sum_{j=1}^8 H(j/2)$ is a reasonable approximation to the average of the temperature function over the interval $[0, 4]$.

- (b) Show that, if we approximate the average temperature with n subdivisions, Prof. Bob Loblaw can write the approximation formula as

$$\frac{1}{n} \sum_{j=1}^n H\left(\frac{4j}{n}\right).$$

- (c) What happens as we take the limit as $n \rightarrow \infty$? Relate this formula to the definition of $\int_0^4 H(t) \, dt$ (remember, we worked this out last time, so you can write down that definition by thinking about adding up the area of n rectangles and then taking a limit to get the exact area, and then double-check that your formula matches what we did last time.)

- (d) Write a formula for the average value of a function on an interval in terms of an integral (instead of a limit of sums).

4. State The Fundamental Theorem of Calculus (both parts).

5. Suppose that $f(x)$ has the following definition:

$$f(x) = \int_0^x (1 + \sin^2 t)^{1/2} \, dt$$

- (a) What is $f'(x)$?

- (b) Express $g(x) = \int_2^{\cos x} (1 + \sin^2 t)^{1/2} \, dt$ in terms of f . Then find $g'(x)$.