1. This problem boils down to "write down the definition of the derivative", but the idea is to fully understand the definition and have an accompanying picture, rather than just memorizing the formula. So try to figure it out by following these steps before looking it up in your book. This approach should help you remember it correctly in the future (like on exams!).

   (a) Given a function $f(x)$, write down a formula for the secant line between the points $(a, f(a))$ and $(x, f(x))$. Sketch a picture of a (curved) function $f(x)$ showing the secant line.

   (b) Using a limit, write a formula for the slope of the tangent line at the point $(a, f(a))$. This is called the derivative of f at a, and is written $f'(a)$.

   (c) We can express the same derivative in a different way. Sketch the same picture as above, but now label the distance between x and a by h. Write an expression for $f'(a)$ in terms of a and h. (Note that you can also let x play the role of a above and get an expression for $f'(x)$ for any x - this is how we usually use the notation in the long run.)

2. Let $f(x) = x^2$.

   (a) Using each of the above definitions, evaluate $f'(0)$ and $f'(1)$ (so you are doing each of these derivatives two separate ways, using the two different versions of the definition of the derivative). Verify on a sketch that these slopes make sense.

   (b) Find an expression for $f'(x)$ that is valid for all values of x.

3. Let $f(x) = \sqrt{3x+2}$.

   (a) Using the definition of the derivative (whichever version you like), evaluate $f'(1)$

   (b) Now find $f'(x)$ for any x (that is in the domain of f).

4. Let

$$f(x) = \begin{cases} -x & : x \leq 0 \\ x^2 & : x > 0 \end{cases}$$

   (a) Is $f(x)$ continuous at $x = 0$?

   (b) Is $f'(0)$ defined? If so, what is it?

5. Let $f(x) = \dfrac{1}{x+3}$.

   (a) Evaluate $f'(2)$

   (b) Find $f'(x)$ for any x (that is in the domain of f).

   (c) If you take the limit of $f'(x)$ as $x \to -3^+$, what do you get? Does this make sense on your picture?

6. Use the Intermediate Value Theorem to prove that the equation $x^2 = 4^x$ has a solution. Hints/steps:

   (a) Write down the statement of the IVT.

   (b) Check if this problem fits the hypotheses of the theorem. Do you need to change anything in the statement of the problem to make it fit?