1. The graph given below represents the derivative of a function f, so the graph is a graph of f'. Where (on which intervals) is the original function f increasing or decreasing? Where is f increasing most rapidly? Copy the graph carefully and sketch the graph of its derivative (that is, the graph of f'').



2. Suppose the dollar cost of producing x video cameras is

$$C(x) = 500x - 0.003x^2 + 10^{-8}x^3.$$

The marginal cost at a level of production x_0 is $C(x_0 + 1) - C(x_0)$, the cost of producing one more video camera if we are already making x_0 cameras. The marginal cost at x_0 can be approximated by $C'(x_0)$ (why? explain). Calculate this and the actual marginal cost at $x_0 = 5000$.

3. As an epidemic spreads through a population, the percentage p of infected individuals at time t (in days) satisfies the equation (called a *differential equation*)

$$\frac{dp}{dt} = 4p - 0.06p^2 \quad 0 \le p \le 100$$

- (a) How fast is the epidemic spreading when p = 10% and when p = 70%?
- (b) For which p is the epidemic neither spreading nor diminishing?
- (c) Plot dp/dt as a function of p.
- (d) What is the maximum possible rate of increase and for which p does this occur?
- 4. Let $f(x) = x^2 e^x$.
 - (a) Find the first four derivatives of f. (Hint: factor out e^x each time you take a derivative.)
 - (b) Guess a formula for $f^{(n)}(x)$.