Instructions: First, we will spend a few minutes discussing the chain rule and doing an example application on the board together. Then, get into 4 groups. Each group will be assigned one of the four problems below. You will spend 40 minutes discussing and doing your problem with your group. Then, you will spend about another 10 minutes thinking about the other 3 problems -for each problem, think about how you would approach it, what parts seem difficult, etc, so that you are prepared to listen to a presentation on it. For the last hour of class, each group will have about 15 minutes to present their problem on the board and explain what the problem was asking and how they solved it and teach it to the class. Remember, you have 40 minutes to get through your problem and be prepared to present it, so try to work efficiently and if you are spending a long time on one part, ask for help. If you finish your problem early, make sure you are prepared to present it, and then start working on one of the other problems. For presentations, each group will get half of one chalkboard - you can start writing on the board ahead of time if you want.

1. Let $S(x)$ =sine of x radians (the usual $sin(x)$ function we've been using).
   Let $G(x)$ = sine of x **degrees**.
   Similarly, let $C(x)$ =cosine of x radians, and let $H(x)$=cosine of x degrees.

   The fact you know from your book is that $S'(x) = C(x)$ and $C'(x) = -S(x)$. You can use this fact in the work below.

   (a) Are S and G the same function? For what values of x is $S(x) = G(x)$? What about $C(x)$ and $H(x)$?
   (b) Let $r(x)$ be the function that takes x degrees and converts the units into radians. Write down what $r(x)$ is as a function of x.
   (c) Express $G(x)$ and $H(x)$ in terms of $S(x)$ and $C(x)$.
   (d) What is $\frac{dG}{dx}$? What is $\frac{dH}{dx}$? (Hint: Use part a) and the chain rule.)
   (e) Express $\frac{dG}{dx}$ and $\frac{dH}{dx}$ in terms of $G(x)$ and $H(x)$. (No mention of sin or S or cos or C allowed.)
   (f) Is it still true that $(G(x))^2 + (H(x))^2 = 1$?
   (g) Why don't we use the unit of degrees in calculus?

2. Imagine a road on which the speed limit is specified at every single point. In other words, there is a certain function L such that the speed limit x miles from the beginning of the road is $L(x)$. Two cars A and B, are driving along this road; car A's position at time t is $a(t)$, and car B's is $b(t)$.

   (a) Assume the car A always travels at the speed limit. What equation expresses that fact? (Hint: the answer is *not* $a'(t) = L(t)$.)
   (b) Suppose that A always goes at the speed limit, and that B's position at time t is A's position at time $t - 1$. Show that B is also going at the speed limit at all times.
   (c) Suppose B always stays at a constant distance behind A. Under what circumstances will B still always travel at the speed limit? (Try to come up with at least one situation where B will always be traveling the speed limit, and then discuss and see if you think of any others. There is at least one simple situation that works.)

3. Let $f(x)$ be a differentiable function. In this problem, you will find a formula for the derivative of $f^{-1}(x)$ that you can use at a point without finding a formula for $f^{-1}(x)$.

   (a) Review the definition of the inverse of a function. How can you explain the concept of an inverse algebraically and graphically? Draw a sketch that shows a standard function and its inverse.

   (b) What does $f \circ f^{-1}(x)$ equal? ($\circ$ stands for function composition, so in general $f \circ g(x) = f(g(x))$).

   (c) Take your equation from the previous part ($f \circ f^{-1}(x) = \_\_$), and use the chain rule to differentiate both sides. Then, solve for $(f^{-1})'(x)$. This gives your formula for the derivative of the inverse of f.

   (d) To see why this is a useful formula, let's do an example where it isn't easy to solve for $f^{-1}(x)$ in general. Let $f(x) = x^3 + x + 4$.

      i. Notice that if you try to set that function equal to y and solve for x, it will not be easily doable. But, you can solve for the value of the inverse at one specific point: find $f^{-1}(6)$ by setting $x^3 + x + 4 = 6$ and solving for x by inspection (that means guess and check easy numbers).

      ii. Find $f'(1)$.

      iii. Use your formula from above to find $(f^{-1})'(6)$.

4. (Section 3.6 # 67 from your book). The number of hours of daylight at any point on Earth fluctuates throughout the year. In the northern hemisphere, the shortest day is on the winter solstice and the longest day is on the summer solstice. At 40 degrees north latitude (about the latitude of Washington, D.C.), the length of a day is given approximately by the function $D(t) = 12 - 3cos(\frac{2\pi(t+10)}{365})$, where D is measured in hours and $0 \leq t \leq 365$ is measured in days, with $t = 0$ corresponding to January 1. (You may use a calculator for the computations in this problem).

   (a) How much daylight is there on March 1 (t=59)?

   (b) Find the rate of change of the daylight function.

   (c) Find the rate at which daylight changes on March 1. Convert your answer to units of minutes per day and explain what this result means.

   (d) At what times of year is the length of the day changing most rapidly? least rapidly?