1. Suppose an object is moving in a straight line according to the formula $s(t)=5 t^{2}+7$. Here the time, $t$, is measured in seconds and the distance traveled, $s$, is measured in meters.
(a) Find the average velocity over the time period from $t=7$ to $t=8$. If you use this as an estimate for the instantaneous velocity at $t=7$, is it an over-estimate or under-estimate?
(b) Find the average velocity for the time period from $t=7$ to $t=7+h$, where $h$ is a constant. (Your answer will be a function of $h$.)
(c) Simplify your expression from part b) so that there is no $h$ in the denominator. Then plug in values $h= \pm 1, h= \pm 0.5$, and $h= \pm 0.1$. What is the physical meaning of the numbers you computed?
(d) What happens to your expression as $h$ becomes very close to zero from both directions? What is the meaning of considering such values of $h$ ?
(e) What is the instantaneous velocity of the object at time $t=7$ ? Make sure to state the correct units.
2. Explain what it means to say that the limit of $f(x)$ as $x$ approaches $c$ is $L$. (This is a bit open ended right now, so discuss it in your own words. Soon you will be given a rigorous definition in lecture, but the intuition is more important right now.)
3. For each of the four cases below, sketch a graph of some function that satisfies the stated condition.
(a) $\lim _{x \rightarrow 2} f(x)=3$ and $f(2)=4$
(b) $\lim _{x \rightarrow 4+} f(x)=5$ and $\lim _{x \rightarrow 4-} f(x)=7$
(c) $\lim _{x \rightarrow 0} f(x)$ does not exist and $|f(x)|<2$ for all $x$
(d) $\lim _{x \rightarrow 0} f(x)=f(0)+1$
4. A function is said to be continuous at the point $x_{0}$ if
(A) $f\left(x_{0}\right)$ is defined
(B) $\lim _{x \rightarrow x_{0}} f(x)$ exists
(C) $\lim _{x \rightarrow x_{0}} f(x)=f\left(x_{0}\right)$
(a) Sketch a graph of a discontinuous function for each of the following:
i. condition $(\mathrm{A})$ holds, but condition $(\mathrm{B})$ does not
ii. condition (B) holds, but condition (A) does not
iii. conditions (A) and (B) both hold, but condition (C) does not
(b) Classify your examples as removable discontinuities, jump discontinuities, or asymptotes.
(c) Could you have drawn examples which would have been classified differently?
