Name: Worksheet 10: Review

Spend 50 minutes doing the following problems. As always, show all your work and reasoning carefully. Your work should be written up well enough so that a classmate can learn how to do the problem by reading what you wrote. There are hints and advice for some problems given in upside down text, so that you can attempt the problems before looking at the hints.

After finishing all the problems, form groups of two or three people and trade your booklets with each other. You will compare your answers and work, and discuss what was explained well, what could have been done differently, and what needs to be fixed. As always, be respectful and constructive in your criticisms.

1. Use the Intermediate Value Theorem to prove that the equation $3^x = x^4$ has a (real) solution.

   Write down the IVT for a general function on a general interval first, and then check that the hypotheses apply in this case. Try to think about what hypotheses are logically needed for the theorem to work.

2. Let $f(x) = x^2 - 3x$.

   (a) Use the definition of the derivative to find $f'(x)$.

   The definition of the derivative has a limit in it. Draw a general sketch to help you remember what the definition should be. Use the power rule to check that your answer is correct.

   (b) Find an equation of the tangent line to the graph of $f(x)$ at the point $x = 2$.

3. Find the derivative of each function. Don't simplify your answer.

   (a) $\cos\big(\sin(x^2+3)\big)$ (b) $e^{2x}\ln\big(\sqrt{x-1}\big)$ (c) $\dfrac{x^2}{csc(x)}$

4. Is $f(x) = \dfrac{x^2 - 3x + 2}{|x-2|}$ continuous? If not, identify where it is discontinuous and classify the discontinuities.

   Write the general definition(s) for a removable or infinite discontinuity in terms of limits.

5. Prove using the (precise) definition of the limit that $\lim_{x\to 3} 2x + 1 = 7$

   First, write the general definition of the derivative in terms of ϵ and δ Use a general sketch to make sure your general definition makes sense.

6. Evaluate the following limits:

   (a) $\lim_{x\to\infty} \dfrac{7x^5 + x^4 + 3}{3x^5 + 5x^2 + x + e}$

   (b) $\lim_{x\to\pi/2} \dfrac{sin(x) - x^2}{cos(x)}$

   (c) $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

   Remember, you should always show your work/reasoning in each step even if the problem doesn't specifically ask you to.