1. Let $S(x)=$ sine of $x$ radians (the usual $\sin (x)$ function we've been using). Let $G(x)=$ sine of $x$ degrees. Similarly, let $C(x)=$ cosine of $x$ radians, and let $H(x)=$ cosine of $x$ degrees.
(a) Are S and G the same function? For what values of $x$ is $S(x)=G(x)$ ? What about $C(x)$ and $H(x) ?$
(b) Express $G(x)$ and $H(x)$ in terms of $S(x)$ and $C(x)$.
(c) What is $\frac{d G}{d x}$ ? What is $\frac{d H}{d x}$ ? (Hint: Use part a) and the chain rule.)
(d) Express $\frac{d G}{d x}$ and $\frac{d H}{d x}$ in terms of $G(x)$ and $H(x)$. (No mention of $\sin$ or S or $\cos$ or C allowed.)
(e) Is it still true that $(G(x))^{2}+(H(x))^{2}=1$ ?
(f) Why don't we use the unit of degrees in calculus?
2. The graph given below represents the graph of $f^{\prime}$, that is, the derivative of a function $f$.
(a) Where (on which intervals) is the original function $f$ increasing?
(b) Where is $f$ increasing most rapidly?
(c) Sketch the graph of $f^{\prime \prime}$.

3. As a certain epidemic spreads through a population, the percentage $p$ of infected individuals at time $t$ (in days) satisfies the equation (called a differential equation)

$$
\frac{d p}{d t}=4 p-0.06 p^{2} \quad 0 \leq p \leq 100
$$

(a) How fast is the epidemic spreading when $p=10 \%$ and when $p=70 \%$ ?
(b) For which $p$ is the epidemic neither spreading nor diminishing?
(c) Plot $d p / d t$ as a function of $p$.
(d) What is the maximum possible rate of increase and for which $p$ does this occur?
4. Consider the equation $x^{2}+y^{2}=1$.
(a) Sketch the graph.
(b) Can you represent that graph as the graph of a single function? Why or why not?
(c) Temporarily pretend $y=f(x)$ is a function of $x$. Rewrite the equation with $f(x)$ in the place of $y$. Differentiate the equation term by term, remembering to use the chain rule when needed. This gives you a new equation.
(d) Solve that new equation algebraically for $f^{\prime}(x)$ (your answer can have an $f(x)$ in it).
(e) Write your formula for $f^{\prime}$ in terms of $x$ and $y$. What does it represent on the graph? What would be a better notation to use here instead of $f^{\prime}$ ?

