Name: Worksheet 12

Instructions: Each group will get half the chalkboard. For each problem, choose a person to be the primary writer. Your group will discuss the problem together, and the writer will write the work up carefully on the board. Other group members can help, but be sure each person gets a turn to be the primary writer. When you have finished writing all your work for one problem on the board, stop and show it to the teacher for comments and discussion. Once the teacher approves, you can erase it and go on to the next problem.

1. Consider the equation $x^2 + y^2 = 1$.
   (a) Sketch the graph.
   (b) Can you represent that graph as the graph of a single function? Why or why not?
   (c) Temporarily pretend $y = f(x)$ is a function of x. Rewrite the equation with $f(x)$ in the place of y. Differentiate the equation term by term, remembering to use the chain rule when needed. This is called implicit differentiation.
   (d) Solve that new equation algebraically for $f'(x)$ (your answer can have an $f(x)$ in it).
   (e) Write your formula for f' in terms of x and y. What does it represent on the graph? What would be a better notation to use here instead of f'?
   (f) Solve the equation $x^2 + y^2 = 1$ for y. (How many functions do you get?). Use this to find a (piecewise) formula for the slope of the tangent line to a circle using explicit differentiation. Show that your answer is equivalent to the one obtained above.

2. Sketch graphs of the relations
   (i) $y^2 - x^2 = 0$
   (ii) $y^2 - x^2 = 1$
   (a) Find all continuous functions defined by (i) and (ii).
   (b) Compute a formula for y' for both (i) and (ii) above.
   (c) Do the functions defined by (i) and (ii) above share a common derivative?

3. Inverse functions review:
   (a) Given a function $f(x)$, define its inverse function. What conditions are needed for the inverse function to exist (i.e. to be a function)?
   (b) Does the function x^2 have a (global) inverse function? Can you restrict its domain so that it does? What about the function $tan(x)$?
   (c) How can you sketch an inverse function graphically, given the graph of the original function? (Hint" try graphing a couple examples.)

4. Let $y = sin^{-1}(x)$. (Notation: note that $sin^{-1}(x)$ defined to be the inverse function of $sin(x)$. It is pronounced "sine inverse" and it is also known as $arcsin(x)$. It is not the same as $1/sin(x)$.)
   (a) Solve the equation $y = sin^{-1}(x)$ for x.
   (b) Use implicit differentiation and the derivative rules we already know to find an expression for $\frac{dy}{dx}$.
   (c) Use trig identities to write your derivative expression in terms of x only.
   (d) Use the same technique to find a general expression for the derivative of an inverse function, that is, set $y = f(x)$, assume f is differentiable and invertible, and find an expression for the derivative of $f^{-1}(x)$.