1. Consider the following functions:
   $g(x) = \dfrac{1}{\cos x + 1}$ on $[0, 2\pi]$
   $h(x) = xe^{-x^2}$
   For each of them,
   (a) find the critical points;
   (b) find the potential inflection points;
   (c) determine intervals on which the function is increasing, decreasing, concave up, and concave down;
   (d) determine which points are maximas, minimas, or inflection points
   (e) sketch the graph.

2. Suppose that a function $f(x)$ is has $f'(x) < 0$ and $f''(x) < 0$ for all x, with $f(3) = 5$ and $f'(3) = -2$.
   (a) Find an integer n with $|f(2) - n| < 1$
   (b) If $f(r) = 0$, find an integer k with $|r - k| < 2$

   (Hints: Think about what the first and second derivative mean on a graph, and sketch a picture showing what you know about the function. If what you are being asked to find looks confusing, remember that when we did epsilon-delta proofs, we discussed several different ways to represent intervals and numbers or variables in them. You may find it helpful to rewrite what you are being asked to find here in a different notation, or in words, or think about what it means on a picture.)

3. Sketch the graph of a function satisfying the following conditions.
   (a) $f'(x) > 0$ and $f''(x) < 0$ for all x.
   (b) $f'(x) < 0$ for $x < 0$ and $f'(x) > 0$ for $x > 0$, and $f''(x) < 0$ for $|x| > 2$ and $f''(x) > 0$ for $|x| < 2$.

4. Water is pumped into a spherical tank of radius R at a variable rate in such a way that the water level rises at a constant rate c. Let $V(t)$ be the volume of water at time t.
   (a) Sketch the graph of $V(t)$ approximately, but with the correct convexity. Where does the point of inflection occur?
   (b) When the water level in the tank is h, the volume of water is given by $V = \pi(Rh^2 - \frac{1}{3}h^3)$. (Recall R is the radius of the tank.) Assume the level rises at a constant rate $c = 1$ (i.e., $h = t$). Find the inflection point of $V(t)$. Does it agree with your sketch?
   (c) Plot $V(t)$ for $R = 1$.